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COHOMOLOGICAL METHODS IN TRANSFORMATION GROUPS

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Preface

It is our aim to give a contemporary account of a small, but well-developed and useful, part of the immense mathematical field of (compact) transformation groups - namely that in which the main tools are ordinary cohomology theory and rational homotopy theory. Furthermore, except for occasional excursions, we shall restrict our attention to those groups for which these methods work best: these are tori and elementary abelian p-groups. (An elementary abelian p-group, or p-torus, where p is a prime number, is just a product of finitely many copies of the cyclic group of order p.) Torus and p-torus actions are of more than mere intrinsic interest, however: one can extrapolate to gain much useful information about more general group actions, often in a way similar to that in which the classification of compact connected Lie groups was achieved by studying the roots and weights associated with representations of maximal tori. Two important references where the reader can see such extrapolation at work are [Quillen, 1971a, b] and [Hsiang, 1975].

Our subject began with the work of P.A.Smith in the 1930s and 1940s; and consequently, it is often called P.A. Smith theory. Important developments were brought together in the Princeton seminar [Borel *et al.*, 1960]. Later the subject received substantial clarification and inspiration, when, prompted by the work of Atiyah and Segal in equivariant K-theory, some ideas, which were implicit in the work of Borel, were reformulated in the succinct Localization Theorem proven independently by W.-Y. Hsiang and Quillen. (Versions of the theorem for generalized cohomology theories were also given by tom Dieck.) Since then one of the most powerful methods to be introduced into the subject has been Sullivan's theory of minimal models, which is particularly useful for studying torus actions by means of rational cohomology and rational homotopy.

Essential contributions, which can now be considered as a classical part of the subject initiated by P.A. Smith, have been made by A. Borel, G. Bredon, P. Conner, E. Floyd, W.-Y. Hsiang, R. Oliver, D. Quillen, J.C. Su, T. Chang, T. Skjelbred and many others. We apologize if we sometimes fail to give the original reference for these results. Recent developments in the area are due to A. Adem, W. Browder, G. Carlsson, D. Gottlieb, S. Halperin and others.

In our mode of presentation we have a dual purpose. On the one hand, part of the book is written as an introduction to the subject for a person with relatively little background who wishes to get the gist of things without undue pain. Thus we have tried to simplify many parts of the book by treating there only G-CW-complexes. This has the advantage that the cohomology theory used is ordinary singular cohomology. We have added exercises in the hope that the book can be used as a textbook as well as for self-study. On the other hand, we want the book to be a useful reference for anyone working on transformation groups; and for this reason in many parts of the book we treat some more general classes of G-spaces. In these more general cases Čech or, equivalently, Alexander-Spanier, cohomology is used.

For most of the book we have tried to keep the prerequisites minimal, assuming only results which can be found easily in standard textbooks. For algebraic topology we refer mainly to [Dold, 1980] and [Spanier, 1966], for homological algebra to [Cartan, Eilenberg, 1956] and [MacLane, 1967], for the general theory of compact transformation groups to [Bredon, 1972] and [tom Dieck, 1987], and for group rings and group cohomology to [Brown, K.S. 1982]. In each case only a small part of the book is required: the reader is not expected to have read them all from cover to cover. We have included an appendix, which summarizes many of the results from commutative algebra which we need; and we have included another appendix, which outlines the homotopy theory of chain complexes. In Chapter 2 we have summarized the main points from rational homotopy theory which are used in the book.

As usual the presentation becomes a little more terse and the prerequisites increase as the book wears on. In Chapter 1, for example, spectral sequences are not used; but from Chapter 3 onward they are used frequently. The spectral sequence which occurs most often is the Leray-Serre spectral sequence of a fibration. For singular theory this can be found in [Spanier, 1966]. For sheaf cohomology, which is equivalent to Čech cohomology for paracompact spaces, it can be found in [Bredon, 1967a]. A useful version can also be found in [Brown, K.S. 1982]. The basic properties of localization of rings and modules are not included in Appendix A: useful references are [Bourbaki, 1961] or [Atiyah, MacDonald, 1969]. The Steenrod algebra, which crops up on a few occasions, can be found in [Spanier, 1966] and [Steenrod, Epstein, 1962].

Since it is our purpose to supplement and not to supersede such references as [Borel, *et al.*, 1960], [Quillen, 1971a, b], [Bredon, 1972], [Hsiang, 1975] and [tom Dieck, 1987], we sometimes quote results from these works without proof. We always include proofs, however, whenever they are essential to the continuity of the text, whenever they are particularly revealing, whenever we can achieve some generalization of existing proofs, or whenever, in our context, existing proofs can be simplified.

A brief first reading of the book might consist of the following: Chapter 1 and Sections 3.1, 3.10, 4.5 and 5.1-5.4. The cohomological part of Section 3.5 could be added easily, and so could be Section 4.6, which is quite self-contained. Armed with the results of Sections 2.1-2.5, one could add Section 3.3, the homotopical part of Section 3.5, and Sections 4.2 and 4.3. For cohomological results involving more general G-spaces one needs Sections 2.6 and 3.4. The main theorem of Section 3.8 can be found quickly and easily in [Hsiang, 1975]; our presentation is more complicated since we treat a non-localized version of the theorem, and we rely on the rather technical Section 3.6. Some of the shorter sections, such as 3.9, 4.7 and 4.8, are not particularly demanding. (In some places we quote results from Sections 3.6 and 3.7 because they are on hand: the reader seeking to avoid these sections can often fill in the arguments without too much difficulty.)

We would like to thank all the people who have helped us with the writing of the book; and, in particular, our heartfelt thanks go to those who have typed the manuscript, mainly Mrs. A. Giese and Mrs. P. Goldstein. We wish to thank also the University of Konstanz, the University of Hawaii and the Deutsche Forschungsgemeinschaft for their support. And we thank Mr. D. Tranah of Cambridge University Press, especially for his much appreciated patience.

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