1

Elastic materials

1.1 Definitions

Elasticity is the property that makes objects spring back to their original shape, after being deformed. A rubber band can be stretched, but when it is released it springs back to its original length. A rubber cushion is squashed flat when I sit on it, but springs back to its original shape when I get up. A bow can be bent, but straightens again when released. Elasticity is most obviously a property of materials like rubber, which can be deformed considerably without breaking, but it is a property of all solids. Steel is elastic, and so also is concrete.

Later chapters in this book are about the functions of elastic materials in the bodies of animals. They show the many ways in which elastic materials are used as springs. This chapter is about the properties of materials, rather than about their functions. A few materials that appear again in later chapters will be used as examples, illustrating some of the methods used to investigate elastic properties and some of the varied properties that are found.

Some terms will be defined by describing an imaginary specimen which (unlike real biological specimens) is perfectly regular and behaves in an ideal way. This ideal specimen is initially a cylinder of length l_0 and uniform cross-sectional area A_0 (Fig. 1.1(*a*)). It is stretched by forces F, acting on its ends, to a new length l (Fig. 1.1(*b*)). As it stretches it gets thinner, and the cross-sectional area of the stretched specimen is A.

The greater the force, the more the specimen stretches. This ideal material obeys Hooke's law, which says that the force is proportional to the extension: a graph of F against $(l - l_0)$ is a straight line through the origin (Fig. 1.1(*c*)). The gradient of the graph is called the tensile stiffness of the specimen (S) and its reciprocal is the tensile compliance (C).

$$S = F/(l - l_0) = 1/C \tag{1.1}$$

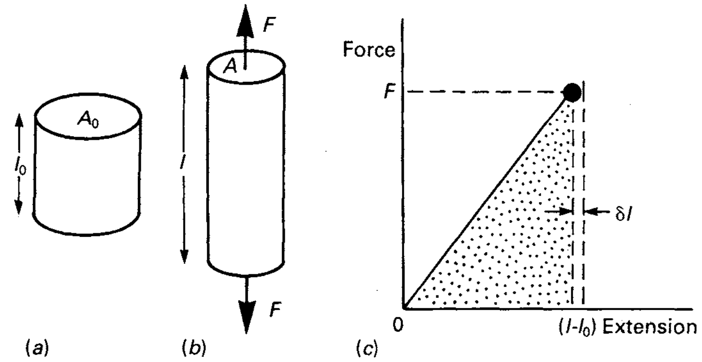
Obviously, a given force will stretch a long specimen more than a short one of the same cross-sectional area, made of the same material. It will stretch a slender specimen more than a stout one of the same initial length. Account is taken of effects like these by calculating stress (force/area) and strain (extension/length).

Here we have to be precise, because stress and strain can each be defined in two different ways:

$$\left.\begin{array}{ll} \text{Nominal or engineering tensile stress} & \sigma = F/A_0 \\ \text{True tensile stress} & \sigma_t = F/A \\ \text{Nominal or engineering tensile strain} & \varepsilon = (l - l_0)/l_0 \\ \text{True tensile strain} & \varepsilon_t = \log_e (l/l_0) \end{array}\right\} \tag{1.2}$$

True stress is truer than nominal stress in a fairly obvious way: the force is divided by the cross-sectional area through which it actually

Fig. 1.1.(*a*) A cylindrical specimen which is stretched, in (*b*), by a force F. (*c*) A schematic graph showing the force plotted against the extension. The stippled area represents the strain energy stored in the stretched specimen.



acts. Similarly, a rather tortuous mathematical argument can make true strain seem truer than nominal strain. However, the nominal values are easier to measure or calculate, and are usually used. Many materials can be stretched only a few per cent or less and for them the nominal and true values are almost identical.

Forces are measured in newtons (N) so stresses are expressed in newtons per square metre, also called pascals (Pa). Strains are ratios of two lengths and so are dimensionless, without units. Young's modulus (E) is tensile stress divided by tensile strain, and is measured in pascals. It is a property of the material, not of the particular structure.

$$E = \sigma/\varepsilon \tag{1.3}$$

Stretching is only one of the ways in which a specimen can be deformed. Another is compression, in which the stress and strain are both negative and can be used as before to calculate Young's modulus. Yet another is shear, the deformation that turns a rectangle into a parallelogram. Fig. 1.2(*a*) represents a thin rectangular slab of material, with a face of area A glued to a rigid wall. In Fig. 1.2(*b*) a force F has been applied to the opposite face, parallel to the wall, and has sheared the slab through an angle θ. This angle (expressed in radians) is the shear strain. The shear stress is F/A. The shear modulus of the material is the shear stress divided by the shear strain, and is one-third to one-half of Young's modulus. (It is one-third of Young's modulus for materials that do not change their volume when they are stretched.)

A specimen that is stretched in one direction is sheared in another.

Fig. 1.2.(*a*), (*b*) Diagrams illustrating shearing. (*c*), (*d*) Diagrams showing that stretching is accompanied by shear. Further explanation is given in the text.

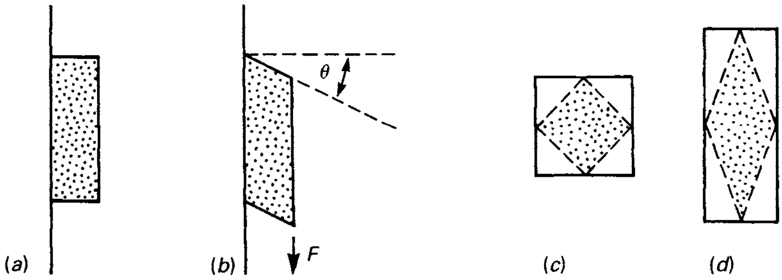


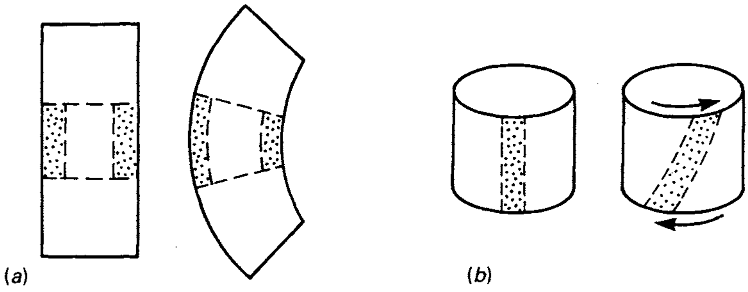
Fig. 1.2(*c*) and (*d*) represent an initially square block of material that is stretched into a rectangle. The block becomes a rectangle, but the stippled square drawn diagonally on it becomes a rhombus. This shows that stretching the block in a vertical direction involves shearing at 45°.

Bending and twisting are other important kinds of deformation. Fig. 1.3(*a*) shows that bending involves extension on the outside of the bend and compression on the inside. Fig. 1.3(*b*) shows that twisting involves shearing around the circumference of the specimen. Mathematical analysis is more complicated for bending and twisting than for stretching and shearing. Other books explain how the stresses and strains can be calculated (for example, Alexander, 1983; Currey, 1984).

A stretched spring or a bent bow has energy (called elastic strain energy) stored up in it. Work is needed to stretch the spring or bend the bow, to supply the strain energy, and most of this work can be recovered in an elastic recoil. In the case of the specimen shown in Fig. 1.1, a force F stretches it to length l. The work needed to stretch the specimen a small additional amount δl (too small to involve an appreciable increase of force) is $F \cdot \delta l$: remember that work is force multiplied by the distance its point of application moves along its line of action. This work $F \cdot \delta l$ is also the area of the narrow strip indicated by the vertical broken lines in Fig. 1.1(*c*). Thus, the work done as the force increases from zero to F is the stippled area under the graph, which is equal to $\frac{1}{2}F(l - l_0)$. This work is converted to strain energy U:

$$U = \tfrac{1}{2}F(l - l_0) \tag{1.4}$$

Fig. 1.3. Diagrams showing (*a*) that bending involves stretching and compression, and (*b*) that twisting involves shear.



By using equation 1.1 to substitute for F or $(l - l_0)$, we can get other useful forms of the equation:

$$U = \tfrac{1}{2}F^2/S = \tfrac{1}{2}S(l - l_0)^2 \qquad (1.5)$$

Alternatively, equation 1.4 can be converted to a different form by using definitions of nominal stress and strain from equation 1.2:

$$U = \tfrac{1}{2}\sigma A_0 \varepsilon l_0 = \tfrac{1}{2}\sigma\varepsilon V \qquad (1.6)$$

where V $(=A_0 l_0)$ is the volume of the specimen.

Most of the remaining sections of this chapter are about the properties of a few examples of biological materials. A good deal of practical detail will be given to show how the properties have been discovered.

1.2 Ligamentum nuchae: properties

The ligamentum nuchae is a highly extensible ligament, composed largely of the protein elastin. It runs along the backs of the necks of hoofed mammals, and helps to support the weight of the head. Its function is discussed in Chapter 2, and it appears here only as an example of a structure whose elastic properties can be investigated extremely easily. Dimery, Alexander & Deyst (1985) performed very simple experiments on the ligamentum nuchae of sheep and deer.

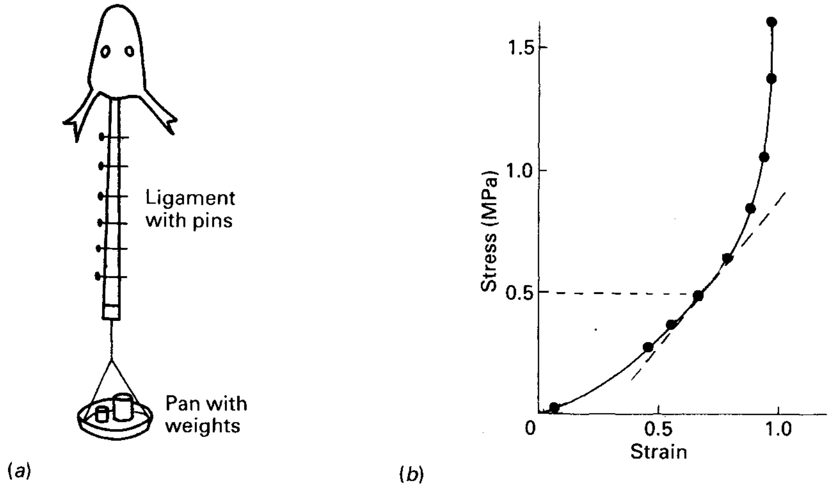
The ligament was dissected out, leaving it attached to the back of the skull. The head was detached from the body and fastened to a firm support, with the ligament hanging down. A pan was tied to the lower end of the ligament so that weights could be put in it to stretch the ligament (Fig. 1.4(*a*)). Care was taken to keep the specimen moist: the properties of biological specimens generally change, if they are allowed to dry.

The ligament tapers (it is most slender near the head) so any load sets up different stresses and strains in its different parts. This difficulty was overcome by sticking pins through the ligament, and measuring the distances between successive pins for each load. Thus, the strain for each segment (from one pin to the next) could be calculated. Afterwards the specimen was cut into segments and the cross-sectional area of each determined, so that stresses could be calculated for each load.

A segment of mass m and density ϱ has volume m/ϱ. Its cross-sectional area is this volume divided by its length l: it is $m/\varrho l$. Thus, cross-sectional areas of ligaments can be calculated from their masses and lengths. This method of measuring cross-sectional areas is often convenient to biologists, but in this particular investigation the volume was measured more directly. A small beaker containing water was placed on a balance, and the reading noted. The segment of ligament, suspended by a fine thread, was lowered into the beaker until it was submerged without touching the glass. This made the reading of the balance increase by an amount equal to the weight of water displaced (by Archimedes' principle). The density of water is 1.00 g cm^{-3}, so an increase of x grammes indicated a volume of x cubic centimetres.

Thus, graphs of stress against strain were obtained (Fig. 1.4(*b*)). If Hooke's law were obeyed, the graph would be straight, with gradient equal to Young's modulus. However, the graph curves. In cases like this it is customary to measure tangent Young's moduli, which are the gradients of tangents to the curve. Fig. 1.4(*b*) gives a tangent

Fig. 1.4.(*a*) A diagram of an experiment on the ligamentum nuchae of a deer (*Capreolus capreolus*). (*b*) A graph of stress against strain, obtained from the experiment. The gradient of the tangent is the tangent Young's modulus at a stress of 0.5 MPa. The data are from the work of Dimery, Alexander & Deyst (1985).



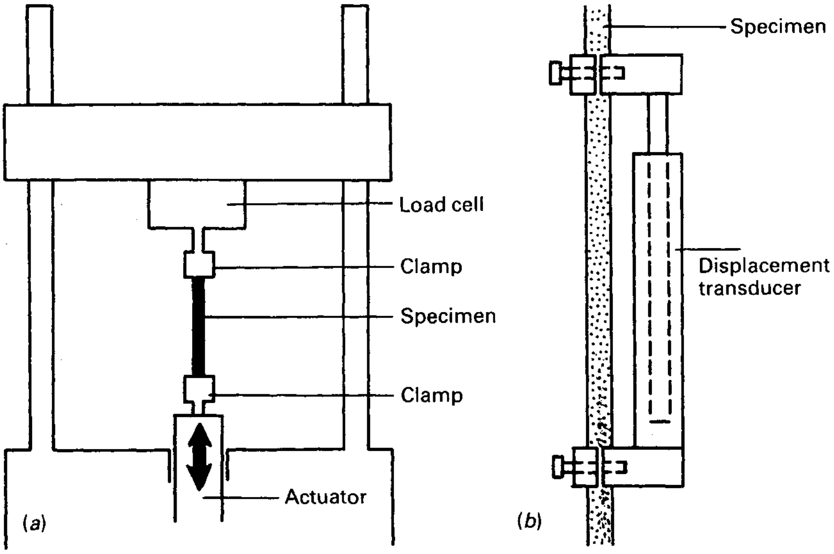
modulus of 1.2 MPa, at a stress of 0.5 MPa, and higher moduli at higher stresses.

1.3 Tendon

Tendon consists largely of the protein collagen. It is much less extensible than ligamentum nuchae: it breaks at strains of about 0.08 (data of Bennett, Ker, Dimery & Alexander, 1986), whereas ligamentum nuchae can be stretched to strains of about 1. This makes it harder to make acceptably accurate measurements of its elastic properties. The experiments on ligamentum nuchae, described in the previous section, were crude but adequate. The experiments on tendon described in this one were much more sophisticated and required immensely more expensive equipment (Ker, 1981).

Tendons were stretched in a dynamic testing machine (Fig. 1.5(*a*)). A length of tendon is held at its ends by clamps. The lower clamp is attached to a hydraulically driven actuator which can

Fig. 1.5. Diagrams of (*a*) a dynamic testing machine, and (*b*) an extensometer.



be made to move down and up as required, stretching the tendon and allowing it to recoil. The upper clamp is attached to a load cell which senses the force on the tendon. Electrical outputs from the machine indicate force and actuator position and can be used to produce records like Fig. 1.6(*a*).

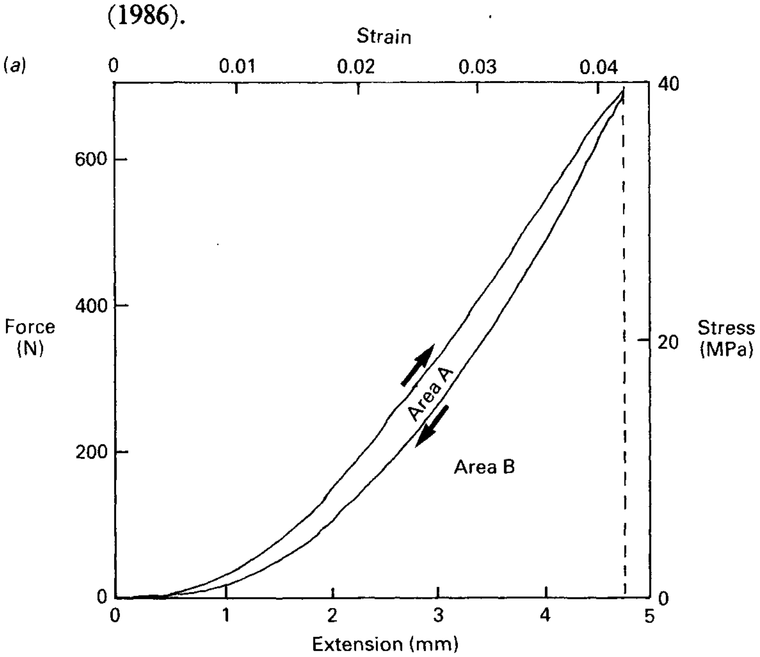
Notice that the record is a loop. The upper line was drawn as the specimen was stretched and the lower line as it shortened. The loop is formed because some of the work done stretching the specimen is degraded to heat instead of being recovered in the recoil. This happens to some extent with all materials but was not apparent in the crude tests on ligamentum nuchae (section 1.2).

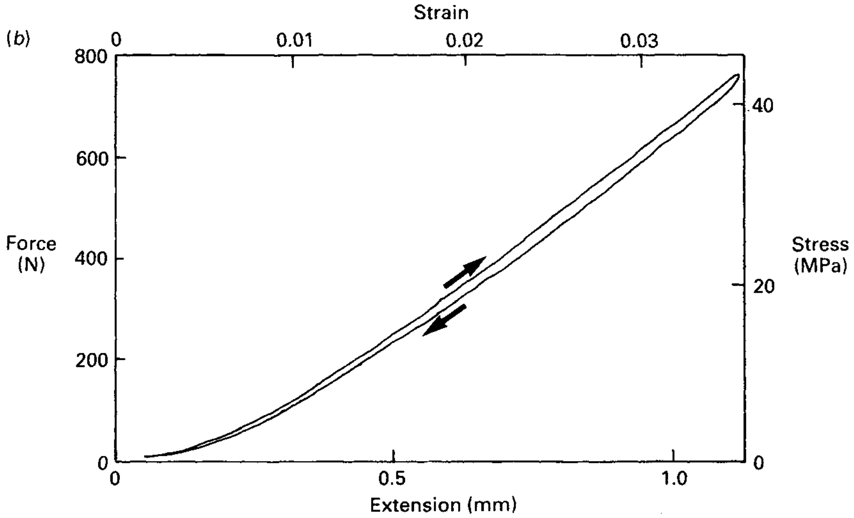
The area $(A + B)$ under the rising line represents the work done stretching the specimen, just as the stippled area in Fig. 1.1(*c*) does. The area B represents the work recovered in the elastic recoil and the loop area (A) the energy lost as heat. There are several confusingly different ways of describing the proportion of energy lost (Ker, 1981). The one used in this book is the energy dissipation $A/(A + B)$. Another one that is commonly used by engineers is the loss tangent, which is about 0.6 times the energy dissipation if both are small.

Fig. 1.6(*a*) was obtained from expensive equipment, but is nevertheless ambiguous and potentially misleading. First, each clamp gripped an appreciable length of tendon (about 20 mm). This makes it impossible to calculate strains accurately, because we cannot be precise about the length of tendon being stretched: where, within the gripped regions, does it effectively end? Secondly, the clamps distorted the tendon severely (as they had to do, to grip it firmly enough). Different parts of the specimen suffered different stresses. Thirdly, energy was used by movement of the gripped region within the clamps, as tension rose and fell. All these effects may be small for very long, slender tendons, such as the tendons of kangaroo tails (Bennett *et al.*, 1986). They are troublesome for most tendons, especially if energy dissipation is to be measured. To eliminate them, we must exclude the distorted regions near the clamps from our measurements of length change. Thus, we cannot use actuator displacement as our measure of length change. We can still use the load cell output to measure the force because the same force acts on every cross-section of the specimen.

Fig. 1.5(*b*) shows an extensometer, a device for measuring length changes in the undistorted part of the specimen (Ker, 1981). It grips

Fig. 1.6. Graphs of force against displacement, obtained when the gastrocnemius tendon of a wallaby (*Macropus rufogriseus*) was tested as illustrated in Fig. 1.5. The actuator moved up and down sinusoidally, with a frequency of 2.2 Hz. In (*a*) force and displacement were both obtained from the outputs of the testing machine. In (*b*) length changes of a shorter segment of the tendon were measured by means of an extensometer. Additional scales show stress and strain. From the data of Ker, Dimery & Alexander (1986).





two points on the tendon lightly, and gives an electrical output that indicates the distance between them. A light grip, which barely distorts the tendon, is adequate because the grips do not have to transmit large forces: they need only be firm enough to overcome the slight friction and inertia of the extensometer.

Fig. 1.6(*b*) shows a record of a test using the extensometer, on the same tendon as Fig. 1.6(*a*). The length changes are smaller because the extensometer spans only part of the distance between the clamps. More significantly, the loop is relatively narrower, representing a smaller energy dissipation, because the losses in the clamps have been excluded.

Fig. 1.6(*b*) shows two interesting characteristics of tendon. First, the energy dissipation is small, about 0.07. This is important for some of the functions of tendon described in later chapters, which require good recovery of energy in elastic recoil. Secondly, the loop curves at low strains but soon becomes almost straight. The tangent modulus is more or less constant (about 1500 MPa, 1.5 GPa) at stresses above 30 MPa. The initial curved ('toe') region of the curve is due to straightening out of slight waviness ('crimp') in the collagen fibrils in the early stages of stretching. This can be seen by observing a fine strand of tendon through a microscope while it is stretched (Diamant *et al.*, 1972).

A tendon is not a solid block of collagen. Rather, it is a bundle of collagen fibres embedded in a jelly-like matrix of much lower modulus. This arrangement makes it flexible, for the same reason that a wire rope is more flexible than a solid steel bar of equal cross-sectional area. It also makes it strong and stiff in one direction only. Transverse forces (which tendons do not normally have to bear) would easily disrupt a tendon by pulling the fibres apart. The matrix consists of mucopolysaccharides (protein–sugar compounds) and water.

1.4 Mesogloea

Sea anemones and jellyfish have a jelly-like layer of mesogloea in their body walls. This is another composite of collagen fibres in a mucopolysaccharide matrix, but the proportion of collagen is much less than in tendon and the fibres run in all directions.