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Lie groupoids and Lie algebroids in Differential Geometry

K. MACKENZIE
Department of Mathematics, University of Melbourne

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CONTENTS

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K. Mackenzie
Frontmatter
More information

INTRODUCTION

The concept of groupoid is one of the means by which the twentieth century reclaims the original domain of application of the group concept. The modern, rigorous concept of group is far too restrictive for the range of geometrical applications envisaged in the work of Lie. There have thus arisen the concepts of Lie pseudogroup, of differentiable and of Lie groupoid, and of principal bundle - as well as various related infinitesimal concepts such as Lie equation, graded Lie algebra and Lie algebroid - by which mathematics seeks to acquire a precise and rigorous language in which to study the symmetry phenomenae associated with geometrical transformations which are only locally defined.

This book is both an exposition of the basic theory of differentiable and Lie groupoids and their Lie algebroids, with an emphasis on connection theory, and an account of the author's work, not previously published, on the abstract theory of transitive Lie algebroids, their cohomology theory, and the integrability problem and its relationship to connection theory.

The concept of groupoid was introduced into differential geometry by Ehresmann in the 1950's, following his work on the concept of principal bundle. Indeed the concept of Lie groupoid - a differentiable groupoid with a local triviality condition - is, modulo some details, equivalent to that of principal bundle. Since the appearance of Kobayashi and Nomizu (1963), the concept of principal bundle has been recognized as a natural setting for the formulation and study of general geometric problems; both the theory of G-structures and the theory of general connections are set in the context of principal bundles, and so too is much work on gauge theory. As an analytical tool in differential geometry, the importance of the principal bundle concept undoubtedly goes back to the fact that it abstracts the moving frame technique of Cartan. An important secondary aim of these notes is to establish that the theory of principal bundles and general connection theory is illuminated and clarified by its groupoid formulation; it will be shown in Chapter III that the Lie theory of Lie groupoids with a given base is coextensive with the standard theory of connections.

To summarize very briefly the work done on groupoids within differential geometry since Ehresmann, there are the following two main areas.

(1) Work on groupoid theory itself. The construction by Pradines (1966, 1967, 1968a,b) of a first-order infinitesimal invariant of a differential groupoid, the Lie algebroid, and his announcement of a full Lie theory for differentiable

groupoids, paralleling the Lie theory of Lie groups and Lie algebras.

Proofs of many of the Lie theoretic results announced by Pradines were given by Almeida (1980); the construction of counterexamples to the integrability of Lie algebroids was announced by Almeida and Molino (1985).

The general theory of differentiable and microdifferentiable groupoids is a generalization of foliation theory, and the techniques used are largely foliation - theoretic in character.

A very recent article on general differentiable groupoids, expanding considerably on Pradines (1966), is Pradines (1986).

(2) Work in which Lie groupoids have been used as a tool or language. Here there is firstly a range of work which may be somewhat loosely described as the theory of Lie equations and Spencer cohomology - see, for example, Ngô Van Quê (1967, 1968, 1969), Kumpera and Spencer (1972) and Kumpera (1975). Secondly, much of the theory of higher-order connections is in terms of Lie groupoids - see Virsik (1969, 1971), Bowshell (1971), and ver Eecke (1981), for example.

Much of this work has also contributed to the theory of differentiable and Lie groupoids per se.

Outside of differential geometry, there are the following major areas.

(3) The work of Brown and a number of co-authors on the theory of general topological groupoids. See Brown and Hardy (1976) and Brown et al (1976).

For references to the considerable body of work by Brown, Higgins and others on multiple groupoid structures and homotopy theory, see the survey by Brown ("Some non-abelian methods in homotopy theory and homological algebra", in Categorical Topology: Proc. Conf. Toledo, Ohio, 1983. Ed. H.L. Bentley et al, Helderman-Verlag, Berlin (1984), 108-146).

(4) Work on the algebraic theory of groupoids, and their application to problems in group theory. See Higgins (1971).

(5) Work on the cohomology of classifying spaces associated with groupoids, usually having non-Hausdorff, sheaf-like topologies. See the survey by Stasheff (1978).

(6) A rapidly growing body of work on the C*-algebras associated with a topological or measured groupoid. See Renault (1980) and Connes ("A survey of

foliations and operator algebras." Proc. Symp. Pure Mathematics, 38 (1), 1982, 521-628. American Mathematical Society, Providence, R.I.).

For the measure theory of groupoids and its use in functional analytic questions see also Seda (1980) and references given there.

A bibliography on all aspects of groupoid theory up to 1976 is given in Brown and Hardy (1976) and Brown et al (1976). The list of references to the present work is not a bibliography.

The primary aim of this book is to present certain new results in the theory of transitive Lie algebroids, and in their connection and cohomology theory; we intend that these results establish a significant theory of abstract Lie algebroids independent of groupoid theory. As a necessary preliminary, we give the first full account of the basic theory of differentiable groupoids and Lie algebroids, with emphasis on the case of Lie groupoids and transitive Lie algebroids. One important secondary aim has already been mentioned – to integrate the standard theory of connections in principal bundles with the Lie theory of Lie groupoids on a given base, to the benefit of both theories. As a matter of exposition, we describe the principal bundle versions of groupoid concepts and constructions whenever this appears to clarify the groupoid theory.

The concept of Lie algebroid was introduced by Pradines (1967), as the first-order invariant attached to a differentiable groupoid, generalizing the construction of the Lie algebra of a Lie group. In the case of Lie groupoids, the Lie algebroid is the Atiyah sequence of the corresponding principal bundle, as introduced by Atiyah (1957). For a differentiable groupoid arising from a foliation, the Lie algebroid is the corresponding involutive distribution. The closely related concept of Lie pseudo-algebra has also been introduced by a number of authors, under a variety of names – see III§2 for references.

In Chapter IV, and in Chapter III§§2,5,7 we undertake the first development of the abstract theory of transitive Lie algebroids and of their connection and cohomology theory. The condition of transitivity for Lie algebroids is related to that of local triviality for groupoids – for example, the Lie algebroid of a differentiable groupoid on a connected base is transitive iff the groupoid is locally trivial. (However that the transitivity condition implies a true local triviality condition for the Lie algebroid is non-trivial – see IV§4.) A transitive Lie algebroid is naturally written as an exact sequence $L \rightarrowtail A \twoheadrightarrow TB$, where TB is the tangent bundle of the base manifold and L is, a priori, a vector bundle whose fibres are Lie algebras; it is, in fact, a Lie algebra bundle.

Exact sequences are generally classified by cohomology in the second degree. Using this point of view, we develop two separate cohomological classifications of transitive Lie algebroids. Firstly, there is a "global" classification in terms of curvature forms and what we propose to call adjoint connections. A transitive Lie algebroid $L \rightarrowtail A \twoheadrightarrow TB$ is characterized by the curvature 2-form $\overline{R}_\gamma: TB \oplus TB \to L$ of any connection $\gamma: TB \to A$ in it, together with the connection ∇^γ in the Lie algebra bundle L induced by γ. Thus, for example, we obtain simple algebraic criteria for a 2-form, with values in a Lie algebra bundle, to be the curvature of a connection in a Lie algebroid. The criteria are a Bianchi identity and a compatibility condition between the given form and the curvature properties of the Lie algebra bundle. At the simplest level, this generalizes the observation that the curvature form of a connection in a principal bundle with abelian structure group must be closed. In cohomological terms this classification is a specialization of the classification of non-abelian extensions of Lie algebroids.

Secondly we give a "local" classification of transitive Lie algebroids by what we propose to call transition forms. These are Lie algebra valued Maurer-Cartan forms. The classification is analogous to that of principal bundles by transition functions, and indeed for a Lie algebroid which is given as the Atiyah sequence of a principal bundle, the transition forms may be obtained as the right-derivatives of transition functions for the bundle. This classification establishes that transitive Lie algebroids are locally trivial in a sense precisely analogous to that true of Lie groupoids. The author obtained this result in 1979 at a time when it was generally believed that all transitive Lie algebroids were the Lie algebroids of Lie groupoids; it is now known that this is not so, and this classification is the more interesting. The key to this result is that a transitive Lie algebroid on a contractible base admits a flat connection, and we obtain this from the cohomological classification of extensions. In turn, the classification of transitive Lie algebroids by systems of transition forms may be regarded as an element in a non-abelian cohomology theory for manifolds with values in Lie algebra bundles, in the same way that the classification of principal bundles by transition functions may be regarded as a cohomological classification.

In §5 of Chapter IV we show that there is a spectral sequence associated in a natural algebraic manner with a transitive Lie algebroid, which generalizes the Leray-Serre spectral sequence for de Rham cohomology of a principal bundle and, in particular, allows coefficients in general vector bundles to be introduced. This algebraization allows the transfer to principal bundle theory of techniques

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Frontmatter
More information

developed for the cohomology of discrete groups and Lie algebras, and we believe it will also provide the correct setting for the study of the cohomology structure of principal bundles with noncompact structure group. Here we only make a beginning on these questions.

In Chapter V we present a cohomological obstruction to the integrability of a transitive Lie algebroid on a simply-connected base. In this case, this obstruction gives a complete resolution of the problem of when a transitive Lie algebroid is the Lie algebroid of a Lie groupoid.

Combining the obstruction to integrability with the global classification of transitive Lie algebroids by curvature forms, we obtain necessary and sufficient conditions for a Lie algebra bundle valued 2-form to be the curvature of a connection in a principal bundle, providing that the base manifold is simply-connected. These conditions generalize and reformulate the integrality lemma of Weil (1958); see also Kostant (1970).

The methods developed in Chapter IV and in Chapter V represent a rather intricate combination of cohomological and connection-theoretic techniques. We believe we have in fact shown that these two subjects are even more inextricably linked, in a nontrivial fashion, than has been realized.

Indeed it should perhaps be emphasized that this is a book about the general theory of connections, since this may not be fully evident from a glance at the table of contents. General connection theory has traditionally taken place on principal bundles, but we argue here that the proper setting for much of connection theory is on a Lie algebroid, and that the relationship between principal bundles and Lie algebroids is best understood by replacing principal bundles by Lie groupoids.

A reader who is interested in the abstract theory of Lie algebroids and/or the integrability obstruction, and who is familiar with principal bundle theory, but does not wish to acquire the Lie groupoid language, could read Chapter III§§2, 5, Appendix A and Chapters IV and V, though they will miss much explanatory material by so doing.

In Chapters I, II and III we give a detailed account of the basic theory of differentiable groupoids and Lie algebroids, with emphasis on the locally trivial case. The presentation is intended to resemble, as far as is possible, the standard treatment of the theory of Lie groups and Lie algebras. Chapter I is an introduction to the algebra of groupoids. In Chapter II we treat topological groupoids, not so much for their own interest - which is considerable - but as a device for setting down the formal content of certain later constructions without the need to address questions of differentiability. Thus - with a few brief exceptions - we address only those matters which have meaning in the differentiable case.

The main business of the book starts in Chapter III and the resemblance between this part of the subject and the standard treatment of Lie groups and Lie algebras will be evident. We first treat questions of differentiability for the constructions of Chapter II and then, in §§2-3, introduce the Lie algebroid of a differentiable groupoid. In §4 we construct the exponential map and use it to compute the Lie algebroids of several important Lie groupoids, central to connection theory. In §6 we establish two of the main results of the Lie theory of Lie groupoids with a given base. In §5 and §7 we present an account of the connection theory of Lie groupoids and transitive Lie algebroids; §5 giving the infinitesimal theory and §7 those aspects which depend on path-lifting or holonomy. In §5 we also begin the classification of transitive Lie algebroids by transition forms.

Much of Chapters I, II and III is the work of other minds. I have given references to the original literature in the text itself, but I have not attempted to write a comparative history. The following features of these chapters are, I believe, new and significant.

——— The construction in II§6 of the monodromy groupoid of a locally trivial topological groupoid, and the proof in III§6 that there is a bijective correspondence between α-connected Lie subgroupoids of a given Lie groupoid, and transitive Lie subalgebroids of its Lie algebroid, and between base-preserving local morphisms of Lie groupoids and base-preserving morphisms of their Lie algebroids.

These results were announced, for general differentiable groupoids and general morphisms, by Pradines (1966, 1967) and proofs in that generality were given by Almeida (1980) and Almeida and Kumpera (1981). The proofs given here make essential use of local triviality to bypass questions of holonomy, and are new and considerably simpler.

——— The circle of ideas concerning frame groupoids of a geometric structure on a vector bundle: The proof of Ngô's theorem III 1.20 by use of III 1.9 - and thus, ultimately, by Pradines' theorem III 1.4; the calculation III 4.7 of the Lie algebroids of isotropy subgroupoids and of the induced representations III 4.8; and the derivation of III 7.11 from these results.

——— The separation of standard connection theory into the infinitesimal connection theory of abstract transitive Lie algebroids (III§5, IV§1) and the path connection theory of locally trivial topological or Lie groupoids (II§7, III§7). The deduction of the Ambrose-Singer theorem (III 7.27) from the correspondence

III 6.1 between α-connected Lie subgroupoids and Lie subalgebroids.

The concept of transition form in III§5, and the results and techniques of Chapters IV and V, have already been referred to above.

Three appendices follow the main text. Appendices B and C are brief summaries of relevant formulas for Lie groups and vector bundles, respectively, and also serve to fix some matters of notation. Appendix A, however, is substantial, and gives a detailed translation of the elementary theory of connections in principal bundles (as given, for example, by Kobayashi and Nomizu (1963) or Greub et al (1973)) into the language of Atiyah sequences. This Appendix is entirely in terms of principal bundles, and makes no use of groupoid concepts. The Atiyah sequence formulation of connection theory has been mentioned in passing by many writers on gauge theory but - to the knowledge of the author - this is the first full account of its equivalence with the usual formulation. Care has been taken with matters of signs, especially since it is necessary to use the right-hand bracket on the Lie algebra of the structure group.

Two major topics have been omitted from these notes. Firstly there is the theory of jet prolongations of differentiable groupoids and Lie algebroids. This is thoroughly treated in existing accounts - see, for example, Kumpera and Spencer (1972), Kumpera (1975) and ver Eecke (1981).

Secondly there is the important body of work revolving around the concept of microdifferentiable groupoid. This is a generalization of the theory of foliations, both in results and techniques. For the construction of the holonomy groupoid of a microdifferentiable groupoid, announced by Pradines (1966), and its applications, see Almeida (1980). Some very brief indications of the results of this theory are included here. The author hopes that this book will also facilitate a wider appreciation of the importance and depth of the general theory of microdifferentiable groupoids. See Pradines (1986) and references given there.

Some nonstandard terminology deserves comment. In I 2.18 and III 2.1 we have used the word "anchor" where Pradines uses "flèche". It seems to us that the English word, "arrow", is overused and colourless. A possible alternative, "transitivity projection", is cumbersome. The anchor ties - or fails to tie - the structure of the groupoid or algebroid to the topology of the base. Secondly, in II 2.22 we use the word "produced" to describe what in principal bundle terms is the bundle $\frac{P \times H}{G}(B,H)$ constructed from a given $P(B, G)$ and a morphism $G \longrightarrow H$. The usual terms "prolongation" and "extension" have other uses in this subject, and "produced" has the virtue of being clearly antonymous to the word "reduced", used to describe the dual concept.

The background needed for this book is slight. A knowledge of the elementary theory of Lie groups and Lie algebras (not including any structure theory), of vector bundles (not including the homotopy classification), and of de Rham cohomology, is essential. Some acquaintance with the theory of connections in principal bundles is desirable, but only so that the purpose of the constructions given here will be clear. For Chapter IV a familiarity with the cohomology theory of either discrete groups of Lie algebras will help, but - as with connection theory - proofs of almost all results are given in full.

This book is designed primarily for those interested in differential geometry. The methods given here are essentially algebraic and since much recent differential geometry is very firmly rooted in analysis, we have given the algebraic constructions in some detail. We feel that the use of algebraic methods to produce cohomological invariants has a substantial history in differential geometry and is capable of much further development.

We use the words 'category' and 'functor' when it is convenient, but we make no actual use of category theory.

In conclusion, there is a point to be made about the need in differential geometry for the general connection theory of principal bundles, as distinct from that merely of vector bundles. So long as one is interested only in geometries with a matrix structure group (that is, in G-structures), the two approaches are, of course, perfectly equivalent. However one of the points of global Lie group theory is that not all Lie groups are realizable as matrix Lie groups (unlike Lie algebras, which always admit faithful finite-dimensional representations), and to work in this generality it is essential to use principal bundles - or Lie groupoids.

Throughout this book we have given most proofs and constructions in considerable detail. In the case of the first three chapters, we have found that even quite simple details can be difficult to supply quickly, on account of the eclectic nature of groupoid theory. In the case of Chapter IV, we have not wanted to presuppose a knowledge of homological algebra. In any case, we believe that there is enough good mathematics to go around, and there seems no reason why anyone should have to do for themselves what the author has done in preparing this book.

This book has been some time in the making. Some of the work recorded here comes from the author's Ph.D. thesis of 1979 written at Monash University and supervised by Dr. Juraj Virsik. I acknowledge with gratitude my debt to Dr. Virsik for proposing the notes of Pradines (1966, 1967, 1968a,b), work well worthy of study. I am also grateful to Professor Jean Pradines, who provided the information which has become III 1.4, and Professor James Stasheff, for detailed stylistic criticism of the material in that thesis in 1979. I am of course myself solely responsible for the form and content of the present book.

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I would like to acknowledge here the work of three authors which has had a profound effect on the overall orientation of the work presented here, but which has seldom had the specific influence which one can acknowledge in text: the work of van Est (1953, 1955a,b) on the cohomology of Lie groups determined the cohomological approach taken in Chapter IV and elsewhere in the author's work; the paper of Kostant (1970) had a seminal influence on the concept of transition form and the construction of the elements e_{ijk} of Chapter V, as well as on IV§3; and the notes of Koszul (1960) first made the author aware of the power of algebraic methods in differential geometry

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