# 1

# Positions of stars

## 1.1 The coordinate system

If we want to study stars, the first thing to look at might be their positions in the sky. This in itself does not tell us much about the nature of the stars, but it is very helpful when we want to find a particular star or a group of stars in the sky. We have to have a reference point with respect to which we can describe the position of the star in which we are interested. We all know or have at least heard about the constellations of stars which in earlier times were extremely helpful in describing the positions of stars with respect to a given star in a particular constellation. We still name the brightest stars according to the constellations in which they are found, but we like to have a more general description of the positions. When looking at the sky we can measure the positions only as projected against the sphere of the sky, i.e., against a two-dimensional surface. We can therefore describe the positions of the stars by two quantities. Since the surface against which we measure the positions is a sphere, we use spherical polar coordinates. Since our telescopes are fixed on the Earth, we use a coordinate system which is fixed with respect to the Earth. The Earth is rotating, but we do not want to have a rotating coordinate system, which would cause many problems. We keep the coordinate system fixed in space. The equatorial plane of our spherical polar coordinate system is identical with the equatorial plane of the Earth, which means that the equatorial plane is perpendicular to the rotation axis of the Earth. Unfortunately, the direction of the rotation axis of the Earth is not fixed in space, but due to the gravitational forces of the sun and the moon on the Earth, the Earth's axis of rotation is precessing, i.e., describes approximately a cone around an axis fixed in the Earth. This causes our reference plane also to precess, which means that the coordinates of the stars are changing in time because the axes of the coordinate system are changing in time, not because the stars are changing their positions. Of course, the stars are also moving in space but that leads to much smaller changes in the coordinates than does the precession of the Earth's rotation axis.

The coordinates which the astronomers use are the right ascension $\alpha$ and the declination $\delta$. The right ascension $\alpha$ corresponds to the longitude, which we use on the Earth's surface to describe the position of a particular place, and the declination $\delta$ corresponds to the latitude which we use on the surface of the Earth, see Fig. 1.1. As we know from the Earth, we still have to define the meridian which we call longitude zero. On Earth this is defined as the meridian which goes through Greenwich. On the celestial sphere we also have to define a meridian through a given point as being longitude or right ascension zero. We could define the position of a given star as right ascension zero, but then that star might turn out to move in space and then that coordinate system would move with this arbitrarily chosen star. We could choose the position of a very distant object, for instance, the position of a quasar. Even a large space motion of such a distant object would not change its position measurably. At the time when the coordinate system was defined the quasars were not known and the distances of other astronomical objects were not known either. The zero point for the right ascension was therefore defined by the direction of a line, namely the line given by the intersection of two planes, the equatorial plane of the Earth and the orbital plane of the Earth around the sun, the ecliptic, see Fig. 1.2. As the orientation of the equatorial plane changes with time because of the precession of the Earth's rotation axis, the direction of the line of intersection of the ecliptic and the equatorial plane also changes with time which means the zero point for the right ascension also changes with time. So the coordinates for all the stars change with time in a way which can be computed from the known motion of the Earth's rotation axis. The right ascension is measured in hours,

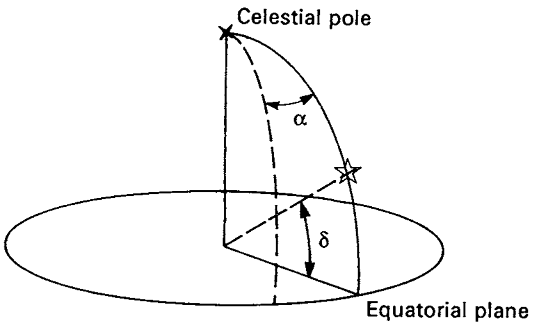


Fig. 1.1. The equatorial plane of the Earth defines the plane for the celestial polar coordinate system, which describes the positions of the stars by right ascension $\alpha$ and the declination $\delta$.

minutes, and seconds. 24 hours correspond to 360 degrees. The right ascension gives the siderial time when the star has its greatest altitude above the horizon. The declination $\delta$ is measured in degrees, $-90° < \delta < +90°$.

There are catalogues which give the coordinates of the stars for a given year, we then have to calculate the corrections to the coordinates for the time, when we want to observe the object. The equations for computing these corrections can be found in Smart's textbook on spherical astronomy (1977). Tables for the corrections are given by Allen (1982).

Catalogues with positions of stars for the year 1855 are, for example, the 'Bonner Durchmusterung' (BD), and for 1900 the 'Henry Draper' Catalogue (HD). Stellar positions for the year 1950 are given in the catalogue of the Smithsonian Astrophysical Observatory (S.A.O.).

## 1.2 Direction of the Earth's rotation axis

From the previous discussion it is apparent that we have to know how the position, or better, the direction of the Earth's rotation axis changes in time. How can we determine this direction? The best way is to take a long exposure photograph of the sky with a fixed orientation of the telescope, preferably close to the direction of the north polar star. Because of the Earth's rotation, which will cause the telescope to change its orientation in space, the stellar positions will apparently move in circles around the direction of the rotation axis during the course of a day, see Fig. 1.3.

Repeated observations of this kind permit a determination of the changing direction of the Earth's rotation axis.

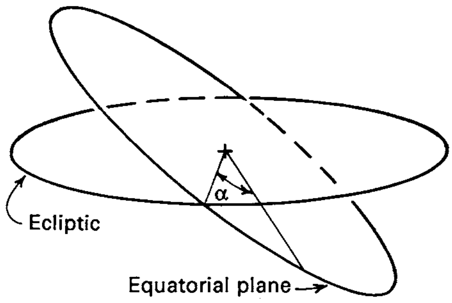


Fig. 1.2. The direction of the intersection between the equatorial plane and the plane of the ecliptic defines the meridian for the zero point of the right ascension. When the orientation of the equatorial plane changes, the position of the zero point meridian changes.

## 1.3 Visibility of the sky

From every point on the surface of the Earth we can see only a fraction of the sky which is determined by our horizon and by the rotation of the Earth, as illustrated by Fig. 1.4. Suppose the observer is at point *P* on the Earth early in the morning. The plane of his horizon is indicated by the solid line. He can observe everything which is above his horizon. Of course, he will only see the stars if the sun is not shining on his side of the Earth. Twelve hours later the observer will be at point P' because of the Earth's rotation

Fig. 1.3. A long-exposure photograph with a fixed position of the telescope pointing towards the North Pole. The positions of the stars describe circles in the sky with the centers of the circles showing the direction of the rotation axis of the Earth. The lengths of the circle segments seen in the photograph are determined by the duration of the exposure. A 12-hour exposure would give a half-circle. (From Abell 1982.)

around the axis $\omega$. The plane of his horizon is now indicated by the dashed line. He can see only what is above this plane. The whole cone, which is cut out by the rotating plane of the horizon is excluded from his view. Only observers at the equator have a chance of seeing the whole celestial sphere during one day, however, they will only be able to see all of the stars during the course of one year, because the sun always illuminates about half of the sky.

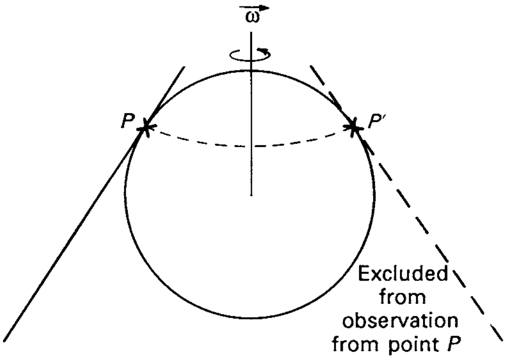


Fig. 1.4. From any point $P$ on the surface of the Earth, a cone is excluded from observation, except for points on the equator.

# 2

# Proper motions of stars

In the previous chapter we have seen that the coordinates of the stars change with time because the coordinate system, which is defined by the rotation axis of the Earth, changes with time. The coordinates of the stars may also change because the stars themselves move in space. Only the motions perpendicular to the direction of the line of sight will actually give a change in the coordinates, see Fig. 2.1. Motions along the line of sight will change the distance but not the coordinates. The motions perpendicular to the line of sight are called proper motions, because they give coordinate changes which are due to the star's proper and not to the Earth's rotation. Velocities along the line of sight are called radial velocities because they go in the direction of the radius of a sphere around the observer. Proper motions are measured by the changes in their right ascension and declination, which are angles. The proper motions are therefore measured in sec or arcsec per year, while the radial velocities are measured by means of the Doppler shift, see Section 9.2, which gives the velocities in km $s^{-1}$. It would be difficult to give proper motions in km/s because the relation between proper motions in arcsec and in km $s^{-1}$ depends on the distance of the star: A given velocity

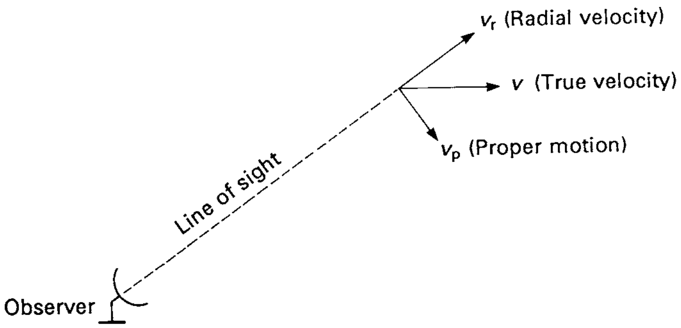


Fig. 2.1. Only motions in the direction perpendicular to the line of sight change the position of the star in the sky, i.e., the point of the projection of the star against the background sphere. Radial velocities do not change the coordinates of the star, only its distance.

of a star in a direction perpendicular to the line of sight will lead to a relatively larger change in position, i.e., to a relatively large proper motion, if the star is nearby and only to a very small change in position if the star is far away, see Fig. 2.2. In fact, proper motion studies are often used to find nearby stars.

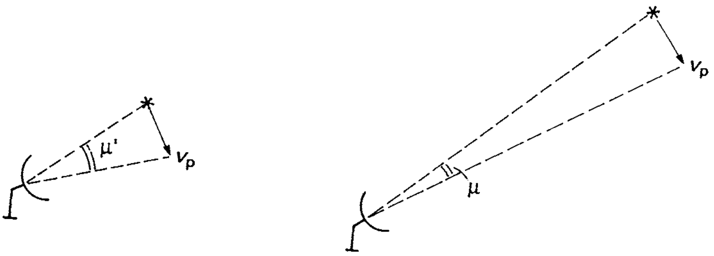


Fig. 2.2. For a given velocity $v_p$ perpendicular to the line of sight the proper motion in arcsec ($\mu'$) is larger for a nearby star than for a more distant star ($\mu$).

# 3

# Distances of nearby stars

## 3.1 The distance of the sun

The distances to nearby objects on Earth are determined by measuring how often a stick of standard length, for instance, a meter stick, fits in between the two objects whose distance we want to measure. For larger distances, this very often does not work. For instance, in a mountain area it would be impossible to measure the distances of two mountain tops in this way.

Our eyes make rough distance determinations without using a meter stick. Our eyes actually use the so-called method of triangulation. For triangulation we observe a given object from two different points whose distance we know, for instance, by measuring with a meter stick.

From two observing points $A$ and $B$ the observed object $C$ will appear projected against the background at different positions $E$ and $D$, see Figure 3.1. For a nearby object there will be a large angle $\gamma$ between the projection points, for an object further away the angle will be smaller, see Fig. 3.2. The relation between the angle $\gamma$ measured from the two points and the distance to the object is given by

$$\sin\left(\frac{\gamma}{2}\right) = a/(2d), \qquad (3.1)$$

where $a$ is the distance between the two observing points, $A$ and $B$, and $d$ is the distance to the object, $C$ or $C'$, from the center of the two observing points,

For large distances we can set $\sin\gamma = \gamma$ if $\gamma$ is measured in radians. (3.1) can then be replaced by

$$\gamma = a/d \qquad (3.2)$$

from which $d$ can be determined if $a$ and $\gamma$ have been measured. For us our two eyes serve as the two observing points, $a$ is then the distance between the eyes.

From (3.2) it is obvious that we can measure larger distances if the baseline $a$ is large, because there is a limit to the size of the angle $\gamma$ which we can still measure. On Earth there is a limit to the length of the baseline $a$, this limit is determined by the diameter of the Earth. It turns out that this baseline is not large enough to measure even the distance to the sun accurately. We can, however, measure distances to nearby asteroids this way and then use Kepler's third law to determine the distance to the sun. We can now also determine the distance to Venus by radar measurements and then again use Kepler's third law to derive the distance to the sun. The method works as fellows: Kepler's third law states that the squares of the orbital periods of the planets are proportional to the third powers of the semi-major axis $b$ of their orbits around the sun, or

$$P^2/b^3 = \text{const.} = A. \tag{3.3}$$

For two planets, for instance, the Earth and Venus (or an asteroid) this tells us that

$$P(\text{Venus})^2/P(\text{Earth})^2 = b(\text{Venus})^3/b(\text{Earth})^3, \tag{3.3a}$$

where $b$(Venus) and $b$(Earth) are the semi-major orbital axis of Venus and Earth. $P$(Earth) is one year and $P$(Venus) is the orbital period of Venus, which is 224.7 days. Equation (3.3a) is one equation for the two semi-major axes of Venus and Earth. If we had one more equation we could determine

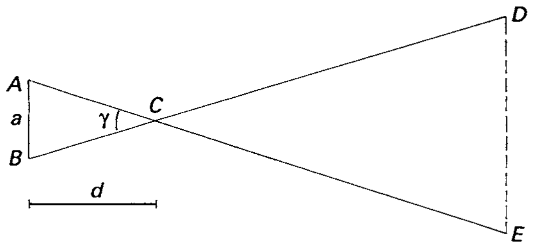


Fig. 3.1. From two observing points $A$ and $B$, distance $a$ apart, the object $C$ is projected against the background at different points $D$ and $E$. The angle $\gamma$ at which the object appears from the observing points $A$ and $B$ is given by (3.1); see text.

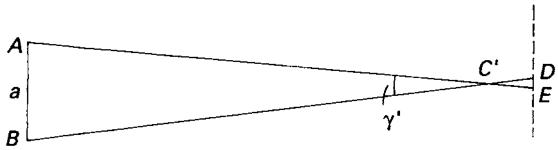


Fig. 3.2. For a distant object $C'$ the angle $\gamma'$ at which the object appears from the observing points $A$ and $B$ is smaller than for a nearby object $C$.

both semi-major axes. The second equation is provided by a measurement of the distance between Venus and Earth at their nearest approach when it can best be measured, see Fig. 3.3. In order to demonstrate the principle, we approximate both orbits by circles (the ellipticities are actually quite small). From Fig. 3.3 we see that then the distance Venus–Earth $d$ is

$$d = b(\text{Venus}) - b(\text{Earth}). \tag{3.4}$$

If we measure $d$, then (3.4) provides the second equation needed to determine both $b$(Venus) and $b$(Earth). Of course, $b$(Earth) is the distance Earth–sun, which we want to determine, and which is generally called one astronomical unit or abbreviated 1 au. The distance Venus–sun is only 0.72 au. At closest approach the distance Venus–Earth is therefore only 0.28 au, see Fig. 3.3. This distance is small enough to be measured by radar. Since the radar signal travels with the speed of light $c$, it takes the radar signal a time $t$ to travel to Venus and back which is given by

$$t = 2\,d/c. \tag{3.5}$$

The time $t$ can be measured and $d$ can be determined from (3.5). Using (3.3a) and (3.4) we then derive the distance Earth–sun to be

$$1\ \text{au} = 1.49 \times 10^{13}\ \text{cm}.$$

Taking into account the ellipticities of the orbits will complicate the mathematics but does not change the principle.

## 3.2 Trigonometric parallaxes of stars

Once we know the orbital diameter of the Earth we can use this length as the baseline, $a$, for further triangulation. If we make one observation on day 1 and the second observation half a year later, we have changed our position in space by the orbital diameter of the Earth. It is not necessary to make the observations from the two observing points at the same time.

From Fig. 3.4 we infer that for a star at the ecliptic pole we will see a change

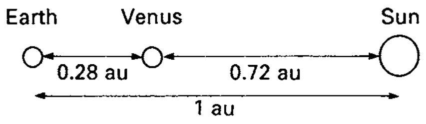


Fig. 3.3. The smallest distance Earth–Venus is only 0.28 times the distance Earth–sun.