Foundations of public economics

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To my father

Contents

Preface and acknowledgments

This book is an attempt to present a variety of important ideas from welfare economics and public finance in a coherent framework and to integrate my own recent research agenda into this framework. Although I hope the book will be suitable for use in graduate level courses, I have not tried to be either fully self-contained or completely comprehensive. Rather, I have tried to give a unified treatment of the main themes in applied welfare economics as I see them; further, I have assumed that the reader is familiar with material covered in a standard first-year microeconomics sequence (roughly at the level of Edmond Malinvaud's *Lectures on Microeconomic Theory* and Hal Varian's *Microeconomic Analysis*) and with the associated mathematical techniques of constrained optimization (as exposited in Michael Intriligator's *Mathematical Optimization and Economic Theory*). Although I have made no conscious effort to differentiate my product from other available sources, I think the reader will find more emphasis here on the expenditure side of public finance than in Anthony Atkinson and Joseph Stiglitz (*Lectures on Public Economics*) and more emphasis on local public issues than in Richard Tresch's *Public Finance: A Normative Theory*.

Many people have given me help in preparing this manuscript. In particular, Thomas Downes, Suzanne Scotchmer, David Wildasin, and several anonymous reviewers have read parts or all of it and provided useful comments. I should also thank classes of students who were subjected to early versions of the manuscript; I hope the final copy is improved as a result of their experience. There probably never would have been a manuscript at all were it not for the Center for Advanced Study in the Behavioral Sciences at Stanford, which provided me with exactly the kind of uninterrupted time necessary for such an endeavor. Further, the manuscript never would have been revised without the efforts of Michael Spence, whose software package *Technical Formatting Program* greatly facilitated the seemingly endless sequence of changes and corrections. Finally, I hope the reader will be able to see the influence of my mentor, Kenneth Arrow. It is no accident that his name is prominent in my reference list – his ideas lie behind many of the topics I discuss.

Notation

Most of the analysis of this book is conducted in a framework where the number of goods, households, clubs, and so on, is arbitrary. Consequently, it is very convenient to employ matrix notation and make full use of the associated vector calculus. Notation can be simplified further if we agree at the outset to certain conventions on vector orientations.

Prices (and "shadow" prices) will be assigned a natural orientation as row vectors, whereas quantities inherit a natural orientation as column vectors. On the rare occasions when we need to change the orientation, transposes will be indicated using curly brackets: $\{\ \}$. Thus, if P is a (row) vector of goods prices, $\{P\}$ is the corresponding column vector. These orientations will extend to functions as well. Thus, if $C(P)$ represents a demand system, it is thought of as a *column vector function.* Correspondingly, if $P(g)$ represents equilibrium prices as functions (say) of levels of public goods provided, it is to be thought of as a row vector function.

Variable subscripts always stand for a matrix index and whenever they are absent, we think of the variable as a vector (or matrix) over the associated index (or indices). Thus, P_i is the price of the ith good. Superscripts stand for matrix labels and will rarely, if ever, be treated as indices of some larger "supermatrix." For example, c^h represents the consumption vector of household h. We will use lowercase letters to represent consumption and production flow variables of *individual economic agents,* reserving the corresponding uppercase variables for economywide aggregates. Thus, C is aggregate consumption ($C = \Sigma_h c^h$). Individual stock variables will have capital letters; for example, B is consumer financial wealth.

When we need to refer to the number of elements in a vector, we do so using the symbol $|\cdot|$ (here, the centerdot is a universal variable). Thus, $|C|$ refers to the number of consumption goods.

We will use the gradient symbol (∇_x) to indicate partial derivatives with respect to a vector x. Again, we require some conventions with respect to orientation. Suppose we have a scalar function f of a vector x. Then the vector of partial derivatives will be denoted $\nabla_x f$. It will have the orientation of a column vector if x is a row vector and vice versa. For example, if $\Gamma(g)$ represents a cost function for producing a public-goods (column) vector g, then $\nabla_g \Gamma$ represents a *row* vector of marginal costs. (Note: frequently we suppress arguments of functions when the meaning is clear.)

Gradients of vector functions naturally form matrices, which will be oriented as follows. Suppose we have a column vector function $C(b)$, where b is a vector of *any* orientation. Then $\nabla_b C$ is the matrix

$$\nabla_b C = \begin{bmatrix} \partial C_1/\partial b_1 & \cdots & \partial C_1/\partial b_{|b|} \\ \partial C_2/\partial b_1 & \cdots & \partial C_2/\partial b_{|b|} \\ \vdots & & \vdots \\ \partial C_{|C|}/\partial b_1 & \cdots & \partial C_{|C|}/\partial b_{|b|} \end{bmatrix}.$$

Observe that orientation has been fixed so that each column corresponds to a partial derivative of the column vector with respect to one argument, and the number of columns equals the cardinality of b. Similarly, we orient the gradient of a row vector function so that each row corresponds to a partial derivative of the row vector.

Note that this last set of rules fails to specify an orientation for the gradient of a scalar function, so we still need the earlier rule specified for that case.

The conventions outlined so far are designed to simplify specific representations encountered in this book. However, they are generally consistent with common practice in a wider arena. Of course, we can apply these rules in series to construct matrices of second- (and higher) order partial derivatives. For example, the Hessian matrix for a scalar function $f(x)$ would be denoted: $\nabla^2_{x,x} f$ (and similarly for other matrices of second partials).

The inner product of matrices x and y will be written simply as xy. Naturally, the matrices must be compatible (x has the same number of columns as y has rows). Thus, if we want to express the inner product of price vector P with itself, we must write $P\{P\}$ (not PP). At a number of points that follow, we will encounter quadratic (and other bilinear) forms, and although all of these can be expressed in terms of the notation developed so far, it is convenient to introduce a separate notation. For example, suppose we encounter a quadratic form involving row vector t and compatible square matrix x; we could always write it as $tx\{t\}$, but instead we choose to express it as $\langle t, x, t\rangle$. More generally, $\langle s, x, t\rangle$ refers to a bilinear form where s has the orientation of a row vector (regardless of its natural orientation), t the orientation of a column vector, and x a matrix of compatible dimensions.

Extensive use will be made of Kuhn–Tucker–Lagrange methods for nonlinear programming. Except where otherwise stated, we will assume all functions are differentiable at least to second order and will work with first- and second-order conditions for optimality. Also, in the interests of notational economy, we will assume interior solutions whenever they seem natural; the reader is invited to replace equalities with the appropriate inequalities and complementary slackness conditions if boundary solutions are contemplated.

Because we want to use the simplest possible inner-product notation, names for variables and parameters must be single letters (otherwise, multiple letters have potentially ambiguous meaning). Even using all uppercase and lowercase Greek and Latin letters, this set of potential names is too small. Hence, we are forced to make two types of compromises.

First, a few letters will have dual meanings. The lists of symbols that follow give a dictionary of meanings with alternative meanings in parentheses. Note that most duplications involve the spatial model on the one hand and the temporal model with uncertainty on the other (these models will never be considered jointly). For example, the letter s is a time index in the latter model whereas it represents a location index in the former; similarly, r represents land rental rates in the spatial model and interest rates in the temporal model.

Second, a few names will be assigned using multiple (uppercase) letters. This will be done only when the associated variables rarely appear in algebraic expressions and have natural meanings as acronyms or words. Thus, for example, VAR and COV are the variance and covariance functions, and RP represents risk premium and TT terms-of-trade effect.

List of symbols: Latin letters

a	generic decision variable (Chapter 12, asset holdings)	A	firm debt
b	government private net inputs	B	bequests, household wealth
c	net individual consumption	C	aggregate net consumption
d	asset dividends	D	government debt
e	exponential symbol	E	expectations operator
f	production functions	F	generic function symbol
g	collective-goods levels	G	congestion levels
h	household index	H	household type
i	generic index	I	income compensation function
j	firm index	J	firm type
k	project capacity (local public finance, index of "active" community)	K	capital stock (size of common)
ℓ	land holdings	L	aggregate land (Lagrangian functions)
m	income levels	M	firm equity
n	population sizes	N	population aggregates (Chapter 7, government effective deficit)
o	status quo reference	O	international prices
p	market parameters	P	consumer prices
q	strategies in mechanisms	Q	producer prices
r	interest rates (land rental rates)	R	individual orderings (aggregate land rents)
s	time index (spatial location)	S	time horizon (Chapter 12, risk-sharing transfers)
t	tax rates	T	"direct" taxes
u	mechanism outcome function	U	direct utility functions
v	asset prices	V	indirect utility functions
w	gradient of welfare function	W	social welfare function
x	variable of integration (planning, social state)	X	Cartesian product (set of social states)
y	private net outputs	Y	aggregate net output
z	exogenous resources	Z	indirect welfare function

List of symbols: Greek letters

α	project parameter		
β	welfare weight		
γ	utility/cost parameters	Γ	costs and cost functions
δ	Taylor's series symbol	Δ	discrete increment
∂	partial derivative symbol		
ϵ	elasticities (regression residual)		
ξ	growth rates (Chapter 9, LM for quantity constraints)		
η	private activity levels		
ψ	pseudoprice of Arrow securities	Ψ	capital funds
κ	various constants		
λ	LM for income	Λ	tax distortion vector
μ	LM for government budget		
ϕ	transport cost	Φ	aggregate transport
ν	LM for profits		
π	firm profits	Π	aggregate profits
ρ	discount factors		
σ	town boundary	Σ	summation symbol
τ	tax function parameters		
χ	cost and type shares		
θ	compensating variation	Θ	naive consumer surplus
ω	state of the world	Ω	shadow values of collective goods
ζ	depreciation rates		

Note: Abbreviation LM means Lagrange multiplier.