

Introduction

Methodology¹

Although a book in mathematical Marxian economics is no longer a unique phenomenon, its author must still confront the opinion held in many circles, both Marxian and non-Marxian, that such an endeavor is a contradiction in terms. Two lines of defense are available: (1) that Marx himself was not against the use of mathematical methods; (2) that regardless of Marx's position, these methods are appropriate to aid in understanding the social phenomena with which Marx was concerned. Although what Marx believed on this question should not settle the issue, if we consider Marxism to be a science and not a religion, it nevertheless appears that Marx was a supporter of the use of mathematical methods in economics. This is shown by the work of Leon Smolinski (1973), who studied Marx's unpublished as well as published manuscripts for his views on the matter. Smolinski reports there was "not a single injunction against mathematical economics [in] Marx's published or unpublished writings." Moreover, Lafargue attributes to Marx the statement: "A science becomes developed only when it has reached the point where it can make use of mathematics" (Smolinski, p. 1201). Still, the opposing circumstantial evidence remains that Marx made very little use of formal mathematics (beyond arithmetic) in his work. As Marx studied algebra and calculus quite extensively in his later years, why did he not use these tools? Two main reasons are suggested by Smolinski: His economic theories had already been formulated before his mathematical studies became intensive, and his mastery of the application of these tools to economic models was very slight. Indeed, Smolinski provides rather incriminating evidence with regard to the second reason, by showing how inept was Marx's effort to analyze the algebraic relationship between the rate of surplus value and the rate of profit in an unpub-

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lished version of *Capital*, Volume III, Chapter III, entitled “The Mathematical Treatment of the Rate of Surplus Value and the Rate of Profit.”²

Regardless of Marx’s position, however, mathematics is a useful tool in Marxian economics. To the extent that abstraction and models are useful, so is mathematics. This would seem self-evident. Several objections nevertheless remain. The most cogent of these seem to be (1) that although mathematics may be useful, it does not accomplish any analytical task that cannot be done without it; (2) that the essential ideas of Marxian social science cannot be mathematized.

Objection 1 does not pass the empirical test of the last century’s research in Marxian economics. The use of mathematical techniques can clarify relationships in an unambiguous way; without these techniques, only intuition can be a guide. But the intuitions of two people may contradict each other: When both are forced to state their beliefs in a common (mathematical) language, there is an objective standard for deciding which is correct. This is seen, in Marxian economics, most clearly in the endless discussions of the transformation problem and the theory of the falling rate of profit. Recent theoretical statements of the problems have resolved many (if not all) of the debates. *After* the mathematics has done its job, it is often possible to state the proof verbally – that is, to avoid the “mathematics.” Hindsight, however, differs from foresight. If a tool acts as a catalyst, and enables us to see how to perform the task for which it was intended, but without its use, more power to it.

A related point to objection 1 is, as Jacob Schwartz (1961) put it, that “mathematics, in doing its good works, has a way of drawing attention to itself.” This is perhaps at the heart of the objection of many Marxists to the use of mathematics. They fear that introduction of the tool distracts attention from the burning social issues of the underlying investigation, and gives the inquiry a gamelike character. This is, no doubt, a danger, but the reciprocal accusation can just as well be made against those Marxists who write pages of “dialectical” reasoning, reveling in the Talmudic play possible in that medium. Any methodology can be abused.

Objection 2 is a more serious point, for certainly mathematics can play only a partial role in Marxian social science, or in any social science, or in any science. Indeed, the essential aspect of a science is confrontation of theory with facts, and mathematics does not produce facts. More specifically, the historical materialist method is central to Marxism, and mathematics does not produce history. The question of the applicability of mathematics, then, must necessarily be limited to

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its role in Marxian social theory, a theory being a way of interpreting historical fact. Here one must distinguish between a theory and its models. In my terminology a theory is not by its nature mathematical. Theories live in an intuitional domain. One tests the consistency of a theory by making models that are schematic representations of the theory and that may use mathematics. A model allows statements to be made that have an undeniable truth value (within the model): Statements made in a theory do not have this logical status. There may be several models of a single theory, some of which verify the theory, others of which nullify it. For example, now that some have produced models of the falling-rate-of-profit (FRP) theory that nullify the theory (as in Chapters 4 and 5 of this book), others are trying to produce models that verify it. If pro-FRP models are successfully produced, they will clearly differ from the anti-FRP models in their assumptions, and such a confrontation will force a more careful refinement of what the underlying theory is. That is, a theory, living in the domain of intuition, necessarily has a certain vagueness. The vagueness is brought into sharp focus by the articulation of contradicting models of the (same) theory.

It is in this sense that “mathematics,” or models, cannot capture all that is contained in a theory. A model is necessarily one schematic image of a theory, and one must not be so myopic as to believe other schematic images cannot exist. Nevertheless, this is not a reason not to use mathematics in trying to understand a theory: for, as has been pointed out above, the production of different and contradicting models of the same theory can be the very process that directs our focus to the gray areas of the theory.

It should be underscored that this discussion applies only to the use of models to test the consistency of a social theory, not the usefulness or accuracy of a theory. The usefulness or accuracy of a theory is tested by confronting it, or its models, with history and data.

Using the distinction that has been stated above between a theory and its models, it is now possible to admit the sense in which objection 2 is viable. Because a social theory, by this definition, is a proposal for ordering and understanding a particular set of historical occurrences and behavior, it does not have the status of a statement in logic. A model, however, contains logical statements. Hence, a model can never replace a theory; that is, a model can never capture every nuance the theory might imply. We might construct models that attempt to capture, schematically, what we conceive of as some of the postulates of the historical–materialist theory, in order to examine whether certain conclusions logically follow from a historical–

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materialist perspective. Nevertheless, no model can ever hope to capture fully the theory of historical materialism. It is as if a theory is an object in an infinite-dimensional space, and a model gives us a projection of the object onto a small subspace. Different models produce different projections; with more models, more projections, we get a more accurate feeling for the implications and the limits of the theory, but we can never capture every dimension of the theory from its models. With sufficient modeling, however, we may come to feel that we have exhausted the interesting content of the theory, and so, for all practical purposes, it is understood.

If one adopts this epistemological posture, it becomes clear that although it is true that models can never entirely capture a theory, that is not a reason not to build models of the theory. Quite the opposite: Models provide perhaps the best way of trying to explore the theory. Thus, although objection 2 is valid in the sense described, it does not follow that mathematical modeling should be abandoned.

We have addressed the issue of whether models can necessarily capture everything contained in theories. Another level of objection 2 is that there are specific concepts of Marxian theories – such as class, power, struggle, consciousness, hegemony, and so on – that are not so amenable to mathematical modeling as are the notions of price, quantity, technology, and so on. This, I think, is not the case. The obvious explanation is that we cannot imagine, perhaps, how to mathematize “class struggle” because no one has ever tried – or, more accurately, there is no social science that has tried for 100 years to do so. But even if we *can* mathematize the notion of, say, class, is there a purpose to it? Here the proponents of objection 2 and objection 1 might unite and say: “Class cannot be mathematized. But even if you can do it, what will be gained in the process? The subtle issues concerning class cannot be clarified with mathematical modeling.”

However, I would argue, as an example, that the inability of Marxists to understand what has transpired in the socialist economies since 1917 is intimately related to an imprecise and vague notion of class. We have a theory of class, and we have understood many dimensions of the theory. But now a historical experience has occurred that exposes our ignorance of the complete theory. That is, there are dimensions of the notion of class that have only become important to our understanding of reality since 1917, and we have little intuition with regard to what is happening in those dimensions. It is time, therefore, to attempt to produce models of the theory of class that can enlighten us as to those hidden dimensions. To be specific: Soviet society has developed in a way that no Marxist would have predicted

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in 1885 or 1917. There is no agreement among Marxists on the extremely fundamental, and apparently simple question: What is the Soviet Union – capitalist, socialist, or something else? To resolve this question, one must understand how to define *class* when the means of production are not privately owned, investment is centrally planned, and so on. In other words, our accepted theory of what constitutes class is exposed as being vague when we try to apply the theory to a new situation – namely, Soviet society. When the vagueness of a theory becomes critical in our ability to use the theory to interpret reality, it is an appropriate time to tighten our understanding of it by constructing models of the theory. Not only do I think that the Marxian notion of class can be modeled, more important, I think that such modeling can illuminate the gray areas of the theory, and enable us to understand the class nature of modern Sovietlike (or, let us say, socialist) societies.

Perhaps a historical parallel is useful here. The prime example of modeling an ethereal concept, in Marxian economics, is the theory of exploitation. Exploitation is a concept like class, power, struggle, consciousness; it might appear to be a concept that mathematics could only reify, but not clarify. Marx wished to understand how exploitation could exist under capitalism, a mode of production in which exchanges are mediated through competitive markets. Under feudalism and slavery, the existence of exploitation posed no riddle because of the overtly coercive mechanism for the expropriation of labor. If one had lived under feudalism, the theory of exploitation as the expropriation of surplus labor would have seemed utterly clear: It is only with the advent of competitive markets that the vagueness became apparent in this theory. How is surplus labor expropriated when there is no *corvée*, but only a labor market? (If you ask the wage worker how long he must work to reproduce his family, he will reply, “Forty hours a week”; if you ask the serf, he will reply, “Only three days for my family, the rest of the time I slave for that jerk on the hill.”) Marx resolved the vagueness in the received theory of exploitation by constructing an essentially mathematical model, and extended the applicability of the exploitation theory to capitalism.

To review this example: Before the advent of capitalism, one might have held that the theory of exploitation necessarily entailed a coercive institution for the exchange of labor. If exploitation means the expropriation of surplus labor, of one class by another, then the historical experience of slave and feudal societies would lead one to believe that a necessary aspect of the theory of exploitation was a coercive institution for labor exchange. Marx observed that this per-

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ception of the theory did not work for capitalism: He wished to claim that exploitation was still occurring, despite the absence of a coercive institution for labor exchange. (Indeed, that is the motivation for Marx's effort to explain the existence of exploitation in the presence of "fair" or competitive markets.) One response to capitalist reality was to say that there could not be exploitation, because markets were competitive. This, in fact, is the essence of the neoclassical approach. Marx's approach was, in the terms of this discussion, to say that our previous perception involved false inferences concerning certain hidden dimensions of the theory. As long as society had experience only with coercive institutions for organizing labor exchange, it was not necessary to understand precisely what the theory looked like along that dimension. Now that history has given us a mode of production with, at least in principle, a noncoercive labor exchange mechanism (a competitive labor market), one must attempt to model the theory to see what it predicts in this new situation. Marx's discovery was that the theory of exploitation works through more mysterious channels than one might have thought. Exploitation is a logically consistent idea even in the presence of competitive markets.

An important task for Marxists today is to extend the theory of exploitation so as to be able to evaluate whether exploitation can exist under socialism. To this end, a model-building, mathematical approach should be as useful as it was for Marx. Indeed, we find ourselves today in a theoretical predicament quite analogous to the one in which Marx found himself. We have a fairly good understanding of the dimension of the theory of exploitation that can be labeled "degree of coerciveness of the institution of labor exchange." The dimension of the theory that we have not had cause to examine, until the last sixty years, is the one labeled "public ownership of the means of production." Just as neoclassical economists are incorrect in assuming that the presence of a labor market means exploitation cannot exist, so Marxists, or more generally, historical materialists, would be incorrect in assuming that the absence of private ownership implies the abolition of exploitation. We lack, however, a model of exploitation under these conditions that can help us evaluate if, in fact, the theory is still applicable under socialism. Judgments on the issue of whether exploitation exists in the Soviet Union differ widely among Marxists, because we have no precise understanding of the relevant dimension of the exploitation theory.

This ends the apology for the mathematical model-building method in Marxian economics. Doubtless most of these arguments

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apply to social science as a whole, and doubtless many have been stated more completely, and with more sophisticated philosophical apparatus, elsewhere. I feel, nevertheless, that it is necessary to include such a statement here.

Two more important methodological issues remain concerning the approach I have taken: the validity of a microfoundations approach in Marxian economics, and the validity of an equilibrium approach.

The *microfoundations* approach consists in deriving the aggregate behavior of an economy as a consequence of the actions of individuals, who are postulated to behave in some specified way. I have taken this approach throughout the book. For example, in the chapters on the falling rate of profit, it is postulated that a technical innovation is introduced only if it increases profits for a capitalist. This micro approach is different from a macro approach, which might say: We postulate that technical change takes the form of an increasing aggregate organic composition of capital. From the micro vantage point, one is not allowed to postulate an increasing organic composition of capital unless one can show what individual entrepreneurial mechanism leads to it. Marxists might question the microfoundations methodology because one of the forceful points of Marx's theory is that the individual is not the relevant unit to examine – it is the class. This might lead one to try and build a model in which classes are the atoms of the system.

I think it should be possible to produce such a model, but I do not believe that model would be contradictory to the ones I have described in this book. The reason is this: That individuals act as members of a class, rather than as individuals, should be a theorem in Marxian economics, not a postulate. Marx's point is that despite the capitalist's incarnation as a human being, he or she is forced by the system to act as an agent for the self-expansion of capital. Workers, similarly, may have their individual yearnings and habits, but conditions of life force them to acquire a class consciousness and to act, at times, as agents of the working class as a whole and not as their own agent. (This might be the situation, for instance, in a strike, where the striker takes great chances for the good of the strike, which are not personally worthwhile.) In each case, Marx has claimed that although people exist organically as individuals, we can *conclude* that they act as members of classes. It is in this sense that class behavior is a theorem and not a postulate of the Marxian theory. [I have discussed this particular theorem of class consciousness elsewhere (Roemer, 1978b and 1979a.)]

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Thus, it is not antithetical to Marxian theory to produce models using the microfoundations approach. In particular, I suggest that one theorem of Marxism is a “microfoundations of class” analysis.

Taking the argument a step further, I would say that it is not only admissible, but important, to take a microfoundations approach in Marxian theory. A common error in Marxian discussions is *functionalism*: to assume that a mechanism necessarily exists to perform actions that must be performed to reproduce the system. Put more simply, if the occurrence of *X* will further the reproduction of capitalist relations, then *X* occurs. For example, if racist attitudes exist among the working class, then capital will be strong. Therefore, capitalism foment racism. What is missing here is a description of the mechanism by which this is accomplished. It may be in the interest of capital as a whole to maintain discriminatory wage differentials for black and white workers of equal productivity, but why should the individual, profit-maximizing capitalist respect this differential when he or she can increase profits by unraveling the differential – that is, by hiring only black workers at a slightly higher wage than they are receiving under the racist regime? [For one answer to this question, see Roemer (1979c).] If we postulate capitalism as a system of anarchic, competitive capitals, each bent on its own expansion, we must face this sort of contradiction from functionalist arguments. Another example comes from some Marxist–radical theories of education. Capitalism does not require a highly educated working class, so the theory goes, but it does require a well-socialized and docile working class. Schools, then, will serve the role of socializing and channeling people into capitalist society, but not of educating them. Now, this conclusion may be true, but the functionalist nature of the argument eclipses the mysterious and difficult part of the phenomenon – how does capitalism ensure this role for schools, when teachers try to teach, students try to learn, and so on? A third example is the role of the state. The capitalist state acts in the interests of the capitalist class – that is the theory. But the theory cannot be convincing unless one can demonstrate the mechanism by which this occurs, especially because capitals do not have a habit of cooperating with each other, as the primary aspect of each capital’s existence is self-expansion and competition against other capitals.

What one requires, then, are microfoundations for the role of racism, education, and the state under capitalism. Other examples abound. In the cases when capitalism is guided as if by an invisible hand to coordinate its preservation in the ways mentioned, one requires an explanation of how anarchic capitals produce such a result.

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A second form of functionalism that exists among Marxists is the converse of the first form: If X has occurred, then X must be in the interests of capital. We again can take an example from education: Because we observe compulsory education, it must be that capital requires such for its reproduction. But this may not be the case: Compulsory education may exist because the working class fought for it. One can find many examples of this form of functionalism in Marxian work: The general consequence of the error is to attribute an omnipotence to the capitalist class that it does not possess in Marxian theory. The capitalist class is pushed along by historical developments: Not everything that happens under capitalism was planned by it, nor is in its best interests. In fact, according to Marx, the general tide of historical development favors the working class. Again, a defense against this form of functionalism is a microfoundations approach.

There is a third form of functionalism among Marxists that, strangely, seems diametrically opposed to the first two forms: If the occurrence of X is necessary for the *demise* of capitalism, then X will come to pass. We can see the general rule of which these different functionalist forms are special cases if we phrase the general functionalist position this way. We postulate a certain outcome for the social system; functionalism then takes the form of claiming that only events occur which lead to that outcome. In the first two forms of functionalism discussed, the outcome is the reproduction of capitalist relations; in the third form, the outcome is the transformation of capitalism into socialism. Perhaps the first two forms of functionalism are short-run variants, and the third form is a long-run variant of the general functionalist interpretation of Marxism.

Examples of the third form of functionalism in Marxian economics are prevalent in crisis theory. The system must have crises, because crisis is necessary for capitalist demise. The rate of profit must fall, because only in this way can crisis be brought about. The working class must become impoverished, because otherwise it will never perform its revolutionary task. Bourgeois democracy must transform itself into fascism, because only fascism will heighten the contradictions of capitalism sufficiently to produce revolutionary transformation, which must occur. These arguments are less than convincing; the form of functionalism they involve is similar to that of the utopian socialism of Marx's time, which postulated socialist transformation without a mechanism. Marx's method was to counter utopian thinking by trying to expose the mechanism that would bring the socialist transformation about.³

Finally, the *equilibrium* method has been used in the models in this

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book. While I have defended the approach of mathematical modeling, and the microfoundations approach, I have less confidence about the equilibrium method. Like many economists of my generation, I am strongly influenced by the power of the equilibrium method: of examining a model when it is at rest, so to speak, in the sense that all the rules that describe how its parts work are simultaneously fulfilled. What is disturbing about the equilibrium method is that it pictures the typical position of the system as a position the system rarely or never enjoys. Of course, no sophisticated economic model builder would claim that economies are in equilibrium in the sense of a static equilibrium model. A model is only an ideal type. However, there seems to be a deep contradiction between using models whose main analytical trick is to postulate a position that is precisely at variance with the most interesting and important aspect of capitalist economy as described by Marxian theory – its incessant, contradictory motion. There is, therefore, the danger that if this intuition is correct, the equilibrium method will prevent one from seeing the most important aspects of the Marxian theory of capital. Knowing no other method, I use the equilibrium method, with the vague thought that, when rereading these pages in twenty years, its obsolescence as a modeling tool for Marxian theory may be clear. (I might add that there is plenty of precedent in Marx's modeling of his theory for the equilibrium method: Consider, for example, the notion of equalization of profit rates among capitals, or the models of balanced growth designed to show that capitalism was capable of reproducing itself.)

Summary

From the preceding discussion, it should now be clear what I am attempting. The goal is to convey my perception of aspects of Marxian economic theory by posing specific models. Furthermore, the aspects of the theory that are discussed are classical ones; there is no attempt to extend the theory to deal with new problems, such as the question of exploitation under socialism discussed above.

In Chapters 1 and 2 the concern is with the Marxian notion of equilibrium, and the theory of exploitation. A definition of equilibrium is proposed that includes not only the usual concept of competition of capital and profit maximization or the accumulation of capital (leading to an equalization of profit rates), but also formalizes the Marxian notion of the reproducibility of the economic system. The model is examined in a general equilibrium framework. Within this framework, exploitation is defined, and the equivalence of exploita-