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0521346541 - Clifford Algebras and Dirac Operators in Harmonic Analysis

John E. Gilbert and Margaret A. M. Murray

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