

INDEX OF NOTATION

Inv A	1	$\forall S, \wedge S$	28
ϕ_A	1	$R(X)$	28
rad A	1	$B(X)$	29
nil A	2	$a \perp b$	29
$\ \cdot \ $	2	$(S(B), \tau_B)$	33
$\sigma(a)$	3	\bar{B}	34
$r(a)$	3	K^*	37
$C(X, \mathbb{C}), C(X)$	4	$=_U, <_U$	37
$ \cdot _S$	4	$[f]_U$	37
ϵ_x	4	$(\mathbb{R}^{\mathbb{N}}/U, <_U)$	38
$Z(f)$	4	$\langle g \rangle$	43
$I(F), J(F)$	5	Z	46
J_x, M_x	5	$\mathcal{L} = \mathcal{L}(\hat{\epsilon}, \hat{=})$	54
$\ell^\infty(\mathbb{C}), \ell^\infty$	5	$\forall, \exists, \hat{=}, \hat{\epsilon}$	54
$ \cdot _{\mathbb{N}}$	5	$\wedge, \vee, \Rightarrow, \Leftrightarrow, \hat{\epsilon}$	56
$\beta\mathbb{N}$	6	$T \vdash \phi$	58
M_p, J_p	6	$\vdash \phi$	59
$c_o(\mathbb{C}), c_o$	6	Con T	60
$c_{oo}(\mathbb{C}), c_{oo}$	6	$\mathbb{M} = (M, E)$	60
N_x	10	$\mathbb{M} \vdash \phi[a_1, \dots, a_n]$	61
K_U	10	$\mathbb{M} \vdash \phi$	62
H_o^∞	21	Th(\mathbb{M})	62
$P(S)$	23	$\mathbb{M} \vdash T$	62
$<_F$	23	$\mathbb{M} \cong \mathbb{N}$	62
$<<$	23	\tilde{a}	71
$<_F$	24	Thm(T)	72
$<<_F$	24	$\ulcorner \phi \urcorner$	77
$B = (B, \wedge, \vee, ', 0, 1)$	26, 130	$\mathcal{L}(f)$	83

$f \beta$	83	$\psi_\epsilon, \psi_-, \psi^*$	156
Ord	84	$\mathfrak{h}_\mathfrak{m}$	157
V_α, V	85	Ord ^V	166
rk(S)	85	$a \triangle b$	173
$S \sim T$	85	x_C	174
$ S $	86	$(Q(\kappa), <), \mathfrak{B}(\kappa)$	177
$\omega, \omega_1, \omega_\alpha$	86, 87	$S^\#$	180
$\aleph_0, \aleph_1, \aleph_\alpha$	86, 87	$a \sim_F b, [a]_F$	187
κ^+	87	$[c]_G, \check{c}/\sim_G$	188
$2^\kappa, \kappa^\lambda$	88	$V^{(w)}$	195
$\mathfrak{c} = 2^{\aleph_0}$	88	$(t)^V$	202
$A < B$	106	e_F, d_ξ^H, D_H, f_x^J	218
$\langle A, B \rangle$	107		
$\langle a_\alpha, b_\beta : \alpha < \kappa, \beta < \lambda \rangle$	107		
$[(f, g)]$	112		
$Q(\langle f_\alpha, g_\alpha \rangle)$	113		
$(\mathbb{R}, <)$	120		
$(\mathbb{Q}, <)$	121		
\mathcal{I}, \mathcal{J}	121		
$\langle A_x, B_x \rangle$	121		
$E_B : (a, b) \mapsto \llbracket a \hat{=} b \rrbracket$	131		
$\sim_B : (a, b) \mapsto \llbracket a \hat{=} b \rrbracket$	131		
$\mathfrak{M}_B = (M, E_B, \sim_B)$	131		
$\llbracket \phi[a_1, \dots, a_n] \rrbracket^{\mathfrak{M}_B}$	132		
$\llbracket T \rrbracket$	133		
$\mathfrak{M}_B(u)$	134		
V_α^B, V^B	137		
$\rho(u)$	137		
$\llbracket u \hat{=} v \rrbracket, \llbracket u \hat{\approx} v \rrbracket,$ $\llbracket u < v \rrbracket$	138		
$V^{(2)}$	138		
u_a	142		
$u \sim v, u \hat{\approx} v$	143		
$\llbracket \phi[u_1, \dots, u_n] \rrbracket$	144		
$b \models \phi[u_1, \dots, u_n]$	145		
$\psi_\wedge, \psi_\vee, \psi_\circ$	155		

INDEX

- AC (= Axiom of choice)
 67,84,147,180,181
- algebra
 Banach 3
 norm 2
 radical 1,4
 regular-open 29,174
- α_1 -field 127
- α_1 -set 119,122,129
- alphabet 54
- anti-isotonic 24,25
- antichain 93,143,209
- Archimedean class 128
- Artin-Schreier 36
- atomic formula 55
- automatic continuity 3,20
- Axiom of Choice 67,79,83,145
 See also AC
- axioms of ZF 147-154
- axioms of ZFC 64-68,79
- Bade and Curtis 8,11,20
- Baire
 category theorem 80,91
 space 91
- Banach algebra 3
- \mathfrak{B} -valued model 131,158
 full 135,158
- Bell and Slomson 79
- β_1 -field 128
- β_1 -set 119,122,124,126,129
- Boolean algebra 26,130
 homomorphism 27
 trivial 27
- Boolean-valued universe 137
- Burgess 182
- canonical well-ordering 138
- Cantor 90
- Cantor set 174
- cardinal 86
 of the continuum 88
 preserved 168
 singular 87,182
- cardinality 85
- ccc (= countable chain
 condition)
 Boolean algebra 93,170,
 175,190,204
 partially ordered set
 93,114
 topological space 92,105
- CH (= Continuum Hypothesis)
 17,76,89,90,93,127,128,130,
 173,176,179,182
- chain 23
- Chang and Keisler 79
- character 1

- choice function 68
- class 84
- cofinality 87
- Cohen 78,174,182
- compatible 29
- complementation 27
- complete
 - Boolean algebra 27
 - partially ordered set 26
 - subalgebra 184
 - theory 72
- completeness theorem 63,130, 160,179,182,228
- completion
 - Boolean algebra 34
 - normed algebra 3
 - partially ordered set 30, 41,210
- Comprehension 65,148
- consistent theory 59,63,73 75,161,162
- Continuum Hypothesis 89
 - See also CH
- cumulative hierarchy of sets 85
- Dales 16
- Dales and Esterle 128
- Davis 79
- Δ -system lemma 104
- Δ_\circ -formula 168,185,207,216
- dense
 - in Boolean algebra 30,187
 - in a partially ordered set 29
- diamond 98
- Dow 127
- embedding 24
- epimorphism 21
- equipotent 85,90
- equivalent
 - formulae 59
 - terms 143
- Esterle 16,21,53
- η_1 -field 108,127
- η_1 -set 107,124,129
- eventually 24
- Extensionality 64,148
- extremely disconnected 29
- field
 - ordered 35,128
 - real-closed 36,127
- filter 31
 - generic 115
 - maximal 31
 - proper 32
- forcing 78,130,144,182
 - argument 162
 - iteration 184
- formula 55
 - atomic 55
 - equivalent 59
- Foundation 66
- Fremlin 103
- full 135
- fullness lemma 145
- gap 107
 - (\aleph_1, \aleph_1) - 112,114,116,129
 - Hausdorff 109,110,115,116, 208
 - (κ, λ) - 112
- Generalized Continuum Hypothesis 89

- generic 93,115
 generic object 187
 Gödel 63,75,76,78,176,182
 Gödel number 77
 Hausdorff 109,129
 Hilbert 75
 homomorphism
 Boolean 87
 discontinuous 7,14,17,18,
 21,39,44,46,47,123,127,183
 radical 8,10,11,14
 hull 5
 incompatible 29
 incompleteness theorem
 first 75,79
 second 75,77,79
 independent 42,73,76,179,227
 infinitesimal 36
 infinite iteration 228
 Infinity 67,153
 integral domain 2
 interpolate 107
 isomorphism
 Boolean 27
 of models 62
 isotonic 24,119
 iterated forcing 228
 iteration theorem 204,228
 Johnson 8,20,127
 join 27
 Kaplansky 7
 Kunen 129,228
 language 54,132
 Lindenbaum algebra 74,79
 logical axiom 57
 logically derivable 58
 MA (= Martin's Axiom)
 80,91,92,93,95,96,100,101,
 102,110,116,123,128,183,
 220,227
 MA' 94,95,223
 main boundedness theorem
 8,12,15
 Martin and Solovay 80,103
 Martin's Axiom 80,91,92,183
 See also MA, MA'
 Martin's Maximum 103
 Matijević^v 77,79
 meet 27
 metatheorem 148,155
 mixing lemma 143
 model 60,127,156
 \mathfrak{B} -valued 131,158
 Boolean-valued 130
 countable 60,63
 extension of 61,78
 non-standard 38
 of ZFC 130,136,156,161
 of ZFC + CH 179
 of ZFC + \neg CH 136,176
 of ZFC + MA + NDH 227
 of ZFC + NDH 216
 theory of 62
 modus ponens 58
 NDH 76,79,127,183,216,220,
 227,228
 nilpotent 2
 nilradical 2
 norm 2,7
 normable 2,17,18,20,128
 order
 divisibility 23,43
 Fréchet 24,109

- inclusion modulo finite sets 23,109
 lexicographic 24,120
 partial 23
 standard 24
 strict partial 22
 strong Fréchet 24,43,109
 total 22
 order-dense 121
 order-isomorphism 24
 order-preserving 24
 ordered field 35,128
 ordinal 82,154
 cofinal subset of 87
 finite 86
 infinite 86
 limit 84
 regular 87
 singular 87
 successor 84
 P-point 17,39,40,42,48
 Pairing 65,149
 partially ordered set 23,209
 cofinal, coinital subset 106
 Power Set 66,150
 prefilter 31,187
 pregap 107
 (\aleph_1, \aleph_1) - 208
 equivalent 107
 incompatible 213
 (κ, λ) - 107,118
 preservation of cardinals 169, 208
 prime ideal 2,17,20,128,129
 proof 58
 Proper Forcing Axiom 103
 radical 1
 homomorphism 8,10,11,14
 rank
 of set 85
 of term 137
 Rasiowa and Sikorski 182
 rational interval 96
 regular-open 28,29,41,174
 Regularity 66,152
 relative consistency 60, 128,182,228
 Replacement 66,151
 Rudin 1,40
 rule
 atomic 55
 connective 55
 of inference 57
 quantifier 55
 satisfaction 61
 scope 56
 Scott 182
 seminorm 2
 seminormable 2,10,14,39, 47,48,53
 semisimple 1
 sentence 57,135
 independent 73,76,77
 separable 17,92
 separative 31,41
 sequence 83
 binary 120
 set
 α_1 - 119,122,129
 basic 70
 β_1 - 119,122,124,126,129
 cardinal of 86
 countable 86

- definable 70
- η_1 - 107,124,129
- rank of 85
- simple 69
- transitive 82
- uncountable 86
- SH (= Souslin's Hypothesis) 98,99
- Shelah 42
- Sierpiński 129
- simple set 69
- Sinclair 53
- singularity
 - point 10
 - set 8,10,11
- Smoryński 79
- Solovay 53,79,182
- Solovay and Tennenbaum 184,228
- Souslin line 98
- spectral radius 3
- spectrum 3
- Stone
 - space 22,33
 - topology 33
- Stone-Čech compactification 6
- subformula 56
- submodel 61
- syntax 55
- term 137
 - for embedding 218
 - for function 180
 - for generic object 187
 - for family of subsets 218
 - for partially ordered sets 218
- theory 57
- complete 72,73
- consistent 59,63,73,75
- inconsistent 59
- model of 62
- transcendence basis 128
- transitive set 82
- truth
 - in a model 61
 - in \mathbb{M}_g 132
 - in V^B 144
 - undefinability of 155
- ultrafilter 32,134,135
 - fixed 35
 - free 35,118,123,128
 - quasi-selective 40
- ultraproduct 38,42,108
- uniform norm 4,7
- Union 65,149
- universe 85
 - Boolean-valued 137
- variable 55
 - bound 57
 - free 56
- weakly seminormable 50,51,53
- well-ordered 81
- well-ordering principle 83
- Woodin's condition 46,53
- Zermelo 84
- zero-set 4
- ZF 68,83,147
- ZFC 1,42,64,68,74,77,127,130,179
- ZFC + CH 17,18
- Zorn's Lemma 26,83