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H. P. Yap

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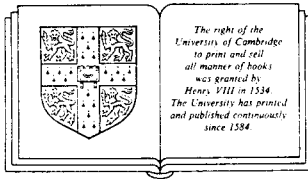
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H.P. YAP
National University of Singapore



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Introduction

A complete, optional course on Graph Theory was first offered to Fourth Year Honours students of the Department of Mathematics, National University of Singapore in the academic year 1982/83. To those students taking this course, it was their first introduction to Graph Theory and so the standard of the course could not be set too high. However, since it was a fourth year Honours Course, the standard could not be too low. For this reason, I decided to use some existing textbooks for the basic results in the first term and to concentrate on only a few special topics in the second term in order to expose the students to some very recent results. This book eventually grew out from the lectures I gave to the students during the academic years 1982/83 and 1983/84.

More than seventy per cent of the materials in this book are taken directly from recent research papers. Each chapter (except chapter 1) gives an up-to-date account of a particular topic in Graph Theory which is very active in current research. In addition, detail of proofs of all the theorems are given and numerous exercises and open problems are included. Thus this book is not only suitable for use as a supplement to a course text at advanced undergraduate or postgraduate level, but will also, I hope, be of some help to researchers in Graph Theory. In fact, Mr. Chen Jing-Hui had written to inform me that by using my lecture notes in his fourth year Graph Theory course in Xiamen University, his students were able to do some research straightway.

A preliminary draft of the manuscript consisting of most of the sections of Chapters 2, 3 and 5 was first submitted to Professor E. B. Davies for consideration to be published as a volume in the London Mathematical Society Student Texts Series in June 1983. This draft was refereed and subsequently transferred to Professor J. W. S. Cassels for consideration to be published as a volume in the London Mathematical Society Lecture Note Series. I am very grateful to the referees for their valuable comments and constructive criticisms, especially for providing a shorter proof of Turner's results on vertex-transitive

graphs of prime order. I am also very grateful to Professors J. W. S. Cassels and E. B. Davies for their encouragement. Thanks are also due to Professor J. W. S. Cassels for assisting me in my application to the National University of Singapore for a 5-month (November 19, 1984 to April 18, 1985) sabbatical leave and to the National University of Singapore for approving my application so that I can concentrate working on this project.

While I was writing this book, I had opportunities to give several survey talks on some topics covered in the book to various institutions in England and China. In November 1982, I visited Oxford University, University of Birmingham and the Open University for two weeks under the sponsorship of the British Council. In May 1984, while on a sightseeing tour of China, I visited East China Normal University (Shanghai), University of Science and Technology (Hefei) and Academia Sinica (Beijing). I also visited University of Cambridge, University of Reading, Oxford University and the Royal Holloway College in January/February, 1985 while I was on sabbatical leave. It is now a great pleasure for me to acknowledge the helpful comments and suggestions from many friends including N. L. Biggs, B. Bollobás, P. J. Cameron, Dong Chun-Fei, A. D. Gardiner, A. J. W. Hilton, E. C. Milner, Wang Jian-Fang, D. J. A. Welsh and R. J. Wilson. I am also indebted to Professor R. M. Weiss for providing a sketch of his nice proof of Tutte's theorem on s -transitive cubic graphs and to my colleague K. M. Koh for helping me in the proofreading. Miss D. Shanthi's excellent word-processing should also be recorded here.

Finally, a few words on the reference system and the exercises of this book. When Theorem $i.j$ is referred to, unless otherwise specified, it is meant that we are referring to Theorem $i.j$ of the same chapter. When an exercise is marked with a minus or a plus sign, it means that the exercise is easy or hard/time-consuming respectively; and if it is marked with a star, it means that it is an open problem or a conjecture.

Contents

Page

Introduction

Chapter 1 Basic Terminology

- | | | |
|-----|-----------------------------|---|
| §1. | Basic graph-theoretic terms | 1 |
| §2. | Groups acting on sets | 4 |

Chapter 2 Edge-colourings of Graphs

- | | | |
|------|---|----|
| §1. | Introduction and definitions | 9 |
| §2. | A generalization of Vizing's theorem | 14 |
| §3. | Critical graphs | 21 |
| §4. | Constructions for critical graphs | 26 |
| §5. | Bounds on the size of critical graphs | 37 |
| §6. | Critical graphs of small order | 43 |
| §7. | Planar graphs | 49 |
| §8. | 1-factorization of regular graphs of high degree | 52 |
| §9. | Applications to vertex-colourings | 56 |
| §10. | Applications to the reconstruction of latin squares | 68 |
| §11. | Concluding remarks | 76 |
| | References | 81 |

Chapter 3 Symmetries in Graphs

- | | | |
|------|---|-----|
| §1. | The automorphism group of a graph | 88 |
| §2. | Asymmetric graphs | 92 |
| §3. | Graphs with a given group | 96 |
| §4. | Vertex-transitive graphs | 99 |
| §5. | Vertex-transitive graphs of prime order | 107 |
| §6. | Auto-extensions | 111 |
| §7. | s-transitive cubic graphs | 115 |
| §8. | 4-ultratransitive graphs | 123 |
| §9. | Hamilton cycles in Cayley graphs | 133 |
| §10. | Concluding remarks | 141 |
| | References | 145 |

	Page
Chapter 4	Packing of Graphs
§1.	Introduction and definitions 156
§2.	Packing $n - 1$ trees of different size into K_n 157
§3.	Packing two graphs of small size 163
§4.	Packing two graphs of order n having total size at most $2n - 3$ 165
§5.	Packing a tree of order n with an $(n, n-1)$ graph 170
§6.	Packing a tree of order n with an (n, n) graph 175
§7.	Packing two $(n, n-1)$ graphs 180
§8.	Packing two graphs of order n having total size at most $2n - 2$ 188
	References 194
Chapter 5	Computational Complexity of Graph Properties
§1.	Introduction and definitions 196
§2.	Some elusive properties; the simple strategy ψ_0 200
§3.	Some non-elusive properties 205
§4.	The diagram of a non-elusive property 209
§5.	The odd-even balanced condition 213
§6.	The Aanderaa-Rosenberg Conjecture 218
§7.	A counterexample to the Rivest-Vuillemin Conjecture 220
§8.	A lower bound for the computational complexity of graph properties 223
	References 226
Index of subjects	228
Index of notation	230