NOTATION INDEX

CONTENTS

ABSTRACT GROUPS

GROUP INVARIANTS

Let G be a group.

h(G) : the Hirsch number of the polycyclic-by-finite group G.

hah(G) : see page 99.

hph(G) : see page 99.

rankG : the (Prüfer) rank of G, viz. the maximum of the minimum number of generators of the finitely generated subgroups of G.

$\pi(G)$: the set of prime divisors of the orders of the elements of G of finite order.

$|G|$: the order of G.

CANONICAL SUBGROUPS & SUBSETS

Let G be a group, α an ordinal number and i a positive integer.

$G^{(\alpha)}$: the α-th term of the derived series of G.
E(G) : see page 71.
F(G) : see page 71.
$F^*(G)$: see page 71.
L(G) : the set of left Engel elements of G (see page 116).
$\bar{L}(G)$: the set of bounded left Engel elements of G (see page 116).
$O_\pi(G)$: the unique maxiaml normal π-subgroup of G.
$O_{\pi,\tau}(G)$: $O_{\pi,\tau}(G)/O_\pi(G) = O_\tau(G/O_\pi(G))$, where π and τ are sets of primes.
R(G) : the set of right Engel elements of G (see page 116).
$\bar{R}(G)$: the set of bounded right Engel elements of G (see page 116).
Z(G) : the Zalesskiĭ subgroup of the soluble group G (see page 188).
$\gamma^i(G)$: the i-th term of the lower central series of G.
$\Delta(G)$: the FC-centre of G (see page 95).
$\zeta_\alpha(G)$: the α-th term of the upper central series of G.
$\zeta(G)$: the hypercentre of G, viz. $\bigcup_\alpha \zeta_\alpha(G)$.
$\eta(G)$: the Hirsch-Plotkin radical of G.
$\eta^{(\alpha)}(G)$: the α-th term of the upper locally nilpotent series of G (see page 99).
$\eta_1(G)$: the Fitting subgroup of G.
$\Lambda(G)$: see page 222.
$\rho(G)$: see page 116.
$\bar{\rho}(G)$: see page 116.
$\sigma(G)$: the Gruenberg radical of G (see page 116).
$\bar{\sigma}(G)$: the Baer radical of G (see page 116).

SPECIAL GROUPS

Let n be a positive integer.

Alt(n) : the alternating group of degree n.
C_n : the cyclic group of order n.
C_∞ : the infinite cyclic group.
GL(n,q) : the general linear group of degree n over the field GF(q).

$O(n,\mathbb{R})$: the n-dimensional real orthogonal group.

Q_{2^n} : the generalized quaternion group of order 2^n.

$Sym(n)$: the symmetric group of degree n.

${}^*X_n(q)$: a group of Lie type (see page 70).

COMMUTATOR NOTATION

Let x, y be elements of the group G.

$[x,_1y] = [x,y] = x^{-1}y^{-1}xy$.

$[x,_ny] = [[x,_{n-1}y],y]$ for $n \geq 2$.

S e T, S |e T : see page 103.

GROUP CONSTRUCTIONS

Let $\{ G_i : i \in I \}$ be a family of groups.

$G_1] G_2$: the split extension of G_1 by G_2, where G_2 acts on G_1.

$G_1 \wr G_2$: a wreath product of G_1 by G_2, being the standard product unless a permutation representation is specified.

$\Pi\, G_i$: the cartesian product of the G_i.

$\times\, G_i$: the direct product of the G_i.

GROUP ACTIONS

Let V be a set and G a group acting on V. Write v^g for the image of $v \in V$ under $g \in G$. Let X be a subset of V and let $v \in V$.

$C_G(X) = \{ g \in G : x^g = x \text{ for all } x \in X \}$.

$C_X(G) = \{ x \in X : x^g = x \text{ for all } g \in G \}$, the set of fixed points of X under G.

$N_G(X) = \{ g \in G : X^g = X \}$.

$v^G = \{ v^g : g \in G \}$.

$\Delta_X(G) = \{ x \in X : x^G \text{ is finite} \}$.

Assume further that V is actually a G-module.

$[V,_1G] = [V,G] = \langle v - v^g : \text{all } v \in V, \text{ all } g \in G \rangle$.

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$[V,_nG] = [[V,_{n-1}G],G]$ for $n \geq 2$.

CALCULUS OF CLASSES

SPECIAL CLASSES

$\underline{A}$: the class of abelian groups.
$\underline{E}_p$: the class of p-groups of finite exponent.
$\underline{F}$: the class of finite groups.
$\underline{F}_p$: the class of finite p-groups.
$\underline{G}$: the class of finitely generated groups.
$\underline{G}_1$: the class of cyclic groups.
$\underline{I}$: the class of locally indicable groups (see page 35).
$\underline{N}$: the class of nilpotent groups.
$\underline{N}_1$: the class of groups all of whose subgroups are subnormal.
$\underline{O}$: see page 26.
$\underline{O}_F$: see page 162.
$\underline{O}_1$: see page 34.
$\underline{O}_2$: see page 37.
$\underline{P}$: the class of polycyclic groups.
$\underline{S}$: the class of soluble groups.
$\bar{\underline{S}}$: see page 82.
$\underline{S}_g$: see page 82.
$\underline{U}$: see page 18.
$\underline{X}$: see page 147.
$\underline{X}_F$: see page 220.
$\underline{Y}_F$: see page 219.
$\underline{Z}$: see page 46.
$\underline{Z}_F$: see page 219.
$\underline{Z}_o$: see page 46.

CLASS OPERATORS

Let $\underline{X}$ be a class of groups.

D : the direct product operator.
L : the local operator.

L_1 : $G \in L_1\underline{X}$ if and only if every cyclic subgroup of G is contained in an $\underline{X}$-subgroup of G.

P : the poly (or extension) operator.

$\acute{P}$: $\acute{P}\underline{X}$ is the class of groups with an ascending series whose factors are $\underline{X}$-groups.

$\grave{P}$: as in $\acute{P}$, with "descending" in place of "ascending".

$\acute{P}_n$: as in $\acute{P}$, but where the series is normal.

P_n : as in $\acute{P}$, but where the series is finite and normal.

Q : the quotient (or homomorphic image) operator.

R : the residual operator.

R_0 : the finite-subcartesian product operator.

S : the subgroup operator.

S_f : the subgroup-of-finite-index operator.

Let A be a unary closure operator.

$\underline{X}^A$: the largest A-closed subclass of $\underline{X}$.

$\underline{X}^{-A}$: the largest A-closed class of groups disjoint from $\underline{X}$.

$\langle A,B\rangle$: the closure operator generated by the operators A and B.

RINGS & FIELDS

Rings except nilrings have a multiplicative identity, which is preserved by homomorphisms.

SUBSETS & INVARIANTS

Let R be a ring, X a subset of R, and $\underline{a}$ an ideal of R.

$\underline{C_R(\underline{a})}$: the set of elements of R that are regular modulo $\underline{a}$ (see page 125).

e(A) : the exponent of the algebra A (see page 48).

J(R) : the Jacobson radical of R.

k(R) : the Krull dimension of R.

$\ell_R(X)$: the left annihilator of X in R.

m(A) : the Schur index of the algebra A (see page 48).

Q(R) : the quotient ring of R, whenever it exists.

$\mathrm{rad}_R X$: see page 141.

rg{X} : the smallest subring of R (with 1) containing X.
R^* : the set of non-zero elements of R.
R_R : R regarded as a right module over itself.
$r_R(X)$: the right annihilator of X in R.
s_n : the standard polynomial identity of degree n.
U(R) : the group of units of R.

SPECIAL RINGS & FIELDS

$\mathbb{A}$: the algebraic closure of the rationals.
$\mathbb{C}$: the field of complex numbers.
$\mathbb{F}_p$: the field of p elements (p a prime).
GF(q) : the field of q elements (q a prime power).
$\mathbb{Q}$: the field of rational numbers.
$\mathbb{Q}_p$: the field of p-adic numbers.
$\mathbb{Q}(\zeta_m)$: the m-th cyclotomic field over the rationals.
$\mathbb{Z}$: the ring of rational integers.
$\mathbb{Z}_p$: the ring of p-adic integers.

RING CONSTRUCTIONS

A_P : see page 49.
A(L) : the universal enveloping algebra of the Lie algebra L.
(R,S,G,H) : the crossed product of S by H (see page 23).

Let S be a ring and G a group.

SG : the group ring of G over S.
$S^\bullet G$: a skew group ring of G over S, some action of G on S being specified (see page 25).

If S is a subring of a ring R and G is a subgroup of U(R) normalizing S, then
S[G] : the subring of R generated by S and G.
S(G) : the division subring of R generated by S and G in situations where R and S are division rings.

FIELDS

Let F be a field.

$\underline{A}(F)$: the quaternion F-algebra (see page 49).
det(x) : the determinant of the matrix x.
deg(f) : the degree of the polynomial f.
$\tilde{F}$: the perfect closure of F.
Br(F) : the Brauer group of F.

Let F be a finite Galois extension of K.

$N_{F/K}(x)$: the norm of the element $x \in F^*$.
$N_{F/K}$: the subgroup of K^* of all $N_{F/K}(x)$ for $x \in F^*$.
Gal(F/K) : the Galois group of F over K.

Assume further that K is an algebraic number field and that $\mathfrak{P}$ is a non-trivial prime ideal of the ring of algebraic integers of K.

$e(\mathfrak{P},F/K)$: the ramification index of $\mathfrak{P}$ in F/K (see page 49).
$f(\mathfrak{P},F/K)$: the residue degree of $\mathfrak{P}$ in F/K (see page 49).
$\mathcal{O}_K$: the ring of algebraic integers of K.
$K_{\mathfrak{P}}$: the completion of K with respect to the valuation determined by $\mathfrak{P}$.

FUNCTORS

Let G be a group, V a G-module and S a ring.

Aut(G) : the group of automorphisms of G.
Aut(S) : the group of automorphisms of S.
Der(G,V) : the group of derivations of G into V.
End(G) : the endomorphism ring of G (whenever G is abelian).
$H^1(G,V)$: the first cohomology group of G with coefficients in V.
Ider(G,V) : the group of inner derivations of G into V.
Inn(G) : the group of inner automorphisms of G.
Inn(S) : the group of inner automorphisms of S.

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Map(X,Y) : the set of all maps from the set X to the set Y.

SKEW LINEAR GROUPS

Let D be a division ring and n a positive integer.

D^n : row n-space over D.
$D^{n\times n}$: the ring of n×n matrices over D.
$\dim_D V$: the dimension of a D-space V over D.
$\mathrm{diag}(X_1,\ldots X_m)$: the set of diagonal matrices with diagonal entries from the appropriate sets X_i.
$(D:E)_r$: the right dimension of D over its division subring E.
GL(n,D) : the group of invertible n×n matrices over D.
SL(n,D) : the special linear group (see page 154).
$Tr_1(n,D)$: the subgroup of GL(n,D) of lower unitriangular matrices.
$Tr^1(n,D)$: the subgroup of GL(n,D) of upper unitriangular matrices.

Let G be a skew linear group and g an element of G.

G^- : see page 81.
G^+ : see page 81.
g_u, g_d : Jordan constituents of g (see page 83).
G_u, G_d : see page 85.
s(G) : see page 21.
u(G) : see page 21.
G^o : the connected component of the identity of the *linear* group G.

SPECIAL SYMBOLS

Let n be an integer.

[n] : the greatest integer less than or equal to n.
-[-n] : the least integer greater than or equal to n.
c(n,q) : see page 113.
e(n,q) : see page 112.
$\beta(n)$: a Jordan function (see page 151).
$\gamma(n,s)$: see page 47.

π' : the set of primes not in the set π.

$\phi(n)$: the Euler function.

ω : the first infinite ordinal.

$\varinjlim$: the direct limit.

$\oplus$: the direct sum.

$\otimes_R$: the tensor product over a ring R.

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