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Skew Linear Groups

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FOREWORD

Our aim in writing this book is to give an up-to-date account of the group-theoretic properties of groups of invertible matrices, where the entries of the matrices lie in a division ring. Our knowledge of this branch of algebra has expanded very rapidly over the last decade. The published part of this material exists only as research papers, and quite a bit, although well-known to the small group of initiates, exists only as private notes. The situation has been reached where, when writing research papers, it is very difficult, or at least very cumbersome, to give the reader adequate references. We hope our book will solve this problem both by providing a textbook for those wishing to study the subject in a systematic way and by providing a convenient reference for research workers. In quite a number of places we have included substantial improvements of both statements and proofs of published theorems.

The importance of linear groups in many branches of mathematics, and even in some parts of physics and chemistry, is well-established. By contrast the much more recent theory of skew linear groups has yet to prove itself. Our final chapter indicates some applications to the theory of group algebras, but even there the intervention of skew linear groups is far from decisive. Thus we feel the need to say a few words here to justify our subject. Our hope of course is that in time many more applications will arise.

Matrix groups over fields are now very well understood (see Wehrfritz [2]) and even if one works over a commutative ring, a very great deal is known (e.g. Wehrfritz [2] Chapter 13 and Wehrfritz [5] Chapter 2). An obvious question is, what happens over a non-commutative ring? One of the first cases to consider is that of a division ring. From the ring-theoretic point-of-view, work of P. M. Cohn especially, shows that, unlike fields, division rings cannot be understood without considering the behaviour of matrices. Thirdly, for group theory, Schur's Lemma shows that every irreducible module over a group G gives rise to a representation of G, usually of infinite degree, over some division ring. Thus representations of groups over division rings are more natural objects of study than representations of groups over fields. The problem is that division rings are usually, or at least presently, too difficult to handle.

PREREQUISITES

We give full, and often alternative, references throughout the book to results that we require from elsewhere, but we indicate here in general terms what we assume of the reader. (We advise readers to start reading the book and to look up what they need as they go along, and not to try and learn the following first.)

We certainly assume a good understanding of algebra up to masters level. Beyond that the requisites for each chapter are very different. Moreover Chapters 2 to 5 inclusive are largely independent of each other.

The first three sections of Chapter 1 are widely used throughout the book. The fourth and final section is used intermittently, mainly to construct counterexamples. Section 1.1 requires little knowledge beyond the Artin-Wedderburn theorems, as does the earlier and main part of Section 1.2. The second half of Section 1.2 uses some Noetherian ring theory. Section 1.3 is in the main very elementary, the exception being 1.3.8 which requires a deep result of H. Heineken and R. H. Bruck. We do make use, in Section 1.3 and elsewhere in the book, of P. Hall's calculus of group classes (see the opening pages of Robinson [1], Vol. 1.). It makes many results very simple to state and clarifies many proofs. The main techniques used in Section 1.4 are Ore domains and Goldie's Theorem. The special case of Goldie's theorem used there we prove in detail.

The remaining chapters make heavier demands on the reader. Chapter 2 utilizes some finite group theory, including a little representation theory, a great deal of number theory in Section 2.1, and a weak version of the classification of the finite simple groups in Sections 2.4 and 2.5. Chapter 3 depends very heavily on the theory of soluble and nilpotent linear groups (Wehrfritz [2]), and to a much lesser extent on the general theory of solubility properties of groups (Robinson [1]). Chapter 4 requires much more ring theory than the preceding chapters. In particluar the structure of group algebras over polycyclic groups, and of modules over such algebras, plays a decisive role throughout the chapter. Our main references here are Passman [2], and for the group theory side, Segal [3]. The very short final Section 4.6 requires some knowledge of Lie algebras. The needs of Chapter 5 are again mainly ring-theoretic, but of a different flavour to those of Chapter 4, and much of what we need we develop in Sections 5.2 and 5.3. In addition some facts about periodic linear groups (see Wehrfritz [2]) are required, as well as the weak version of the classification of finite simple groups. The very brief Chapter 6 shows how some of the earlier results, especially those of Chapter 5, shed light on certain aspects of the theory of group algebras.

The first author wishes to thank Queen Mary College for their hospitality during his stay there. Further, we would like to thank Angela Ridealgh, Marie Fairbrass and Ann Cook, for their cheerful and uncomplaining struggles with illegible manuscripts.

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