

## CONTENTS

<i>Preface to the second edition</i>	page vii
<i>Preface</i>	ix
<b>1 GROUP REPRESENTATIONS</b>	<b>1</b>
1.1 Introduction	1
1.2 $G$ -modules	5
1.3 Characters	9
1.4 Reducibility	10
1.5 Permutation representations	17
1.6 Complete reducibility	20
1.7 Schur's Lemma	24
1.8 The commutant (endomorphism) algebra	27
<b>2 ELEMENTARY PROPERTIES OF GROUP CHARACTERS</b>	<b>37</b>
2.1 Orthogonality relations	37
2.2 The group algebra	42
2.3 The character table	49
2.4 Finite Abelian groups	52
2.5 The lifting process	57
2.6 Linear characters	62
<b>3 INDUCED CHARACTERS</b>	<b>69</b>
3.1 Induced representations	69
3.2 The reciprocity law	73
3.3 The alternating group $A_5$	75
3.4 Normal subgroups	79
3.5 Tensor products	83
3.6 Mackey's Theorem	94
<b>4 PERMUTATION GROUPS</b>	<b>101</b>
4.1 Transitive groups	101
4.2 The symmetric group	106
4.3 Induced characters of $S_n$	108

## CONTENTS

4.4	Generalised characters	110
4.5	Skew-symmetric polynomials	111
4.6	Generating functions	114
4.7	Orthogonality	116
4.8	The degree formula	120
4.9	Schur functions	123
4.10	Conjugate partitions	128
4.11	The characters of $S_5$	135
5	GROUP-THEORETICAL APPLICATIONS	141
5.1	Algebraic numbers	141
5.2	Representations of the group algebra	146
5.3	Burnside's $(p, q)$ -theorem	148
5.4	Frobenius groups	152
6	ARITHMETIC PROPERTIES OF GROUP CHARACTERS	160
6.1	Real character values	160
6.2	Rational character values	161
6.3	A congruence property	165
7	REAL REPRESENTATIONS	170
7.1	Statement of the problem	170
7.2	Quadratic forms	171
7.3	Orthogonal representations	173
7.4	Bilinear invariants	177
7.5	The character criterion	181
	APPENDIX	186
A.1	A generalisation of Vandermonde's determinant	186
A.2	The alternant quotient	188
A.3	Jacobi's Theorem on inverse matrices	191
A.4	Quadratic forms	194
A.5	Congruence relations in an algebraic number field	197
	<i>List of character tables</i>	205
	<i>Solutions</i>	206
	<i>Bibliography</i>	224
	<i>Index</i>	226