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978-0-521-33781-6 - Introduction to Group Characters, Second Edition

Walter Ledermann

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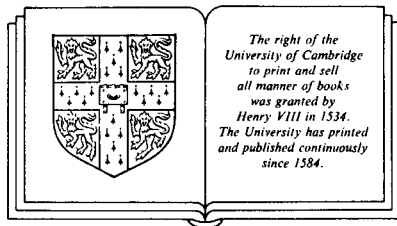
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WALTER LEDERMANN



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PREFACE TO THE SECOND EDITION

I was pleased to accept the publisher's invitation to prepare an enlarged second edition of my book.

The theory of group representations is a large field, and it was difficult to decide what topics should fill the additional space that had become available.

Several colleagues were kind enough to give advice and also to draw my attention to errors in the first edition. I am particularly grateful to A. A. Bruen, R. W. Carter, W. L. Edge and G. D. James for their valuable comments.

However, it was impossible to incorporate all their suggestions. A selection had to be made: in accordance with the principal aim of this text, I have continued to place the emphasis on group characters rather than on the underlying representations.

The new material includes further work on tensor products, arithmetical properties of character values and the criterion for real representations due to Frobenius and Schur.

Once again, I should like to express my appreciation to the officers of Cambridge University Press for their help and understanding during the somewhat lengthy process of completing my task.

W. Ledermann

April, 1986

PREFACE

The aim of this book is to provide a straightforward introduction to the characters of a finite group over the complex field. The only prerequisites are a knowledge of the standard facts of Linear Algebra and a modest acquaintance with group theory, for which my text [13] would amply suffice. Thus the present volume could be used for a lecture course at the third-year undergraduate or at the post-graduate level.

The computational aspect is stressed throughout. The character tables of most of the easily accessible groups are either constructed in the text or are included among the exercises, for which answers and solutions are appended.

It goes without saying that a book on group characters must begin with an account of representation theory. This is now usually done in the setting of module theory in preference to the older approach by matrices. I feel that both methods have their merits, and I have formulated the main results in the language of either medium.

In this book I confine myself to the situation where representations are equivalent if and only if they have the same character. As soon as this fundamental fact is established, the emphasis shifts from the representations to the characters. Admittedly, some information is thereby sacrificed, and I had to be content with somewhat weaker versions of the theorems of A. H. Clifford [4] and G. W. Mackey [15]. However, character theory is sufficiently rich and rewarding by itself, and it leads to the celebrated applications concerning group structure without recourse to the underlying representations.

In the same vein, I have concentrated on the characters of the symmetric group rather than on its representations. The latter are expounded in the monographs of D. E. Rutherford [20] and G. de B. Robinson [19]. The cornerstone of our treatment is the generating function for the characters, due to Frobenius [9], whence it is easy to derive the Schur functions and their properties. On returning to Frobenius's original memoirs after many years I came to realise that familiarity with recondite results on determinants and symmetric functions that were common knowledge around 1900, could no longer be taken for granted in our

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PREFACE

time. I therefore decided to expand and interpret the classical masterpieces so as to make them self-contained without, I trust, spoiling the flavour of the creative power that permeated the early writings on this subject. In order to avoid unduly long digressions I relegated some of the auxiliary material to the Appendix.

There is a fairly extensive literature on representation theory, to which the reader may wish to turn for further instruction. Some of these books are listed in the Bibliography (p. 224). The substantial works of C. W. Curtis and I. Reiner [5] and L. Dornhoff [7] contain excellent bibliographies, which I do not wish to duplicate here. D. E. Littlewood's treatise [14] furnishes a great deal of valuable information, notably about the symmetric group.

My own interest in the subject goes back to an inspiring course by Issai Schur which I attended in 1931. This was subsequently published in the 'Zürich Notes' [21a]. Occasionally, Schur would enliven lectures with anecdotes about his illustrious teacher Frobenius, and I may be forgiven if I have succumbed to a bias in favour of an ancestral tradition.

My thanks are due to the Israel Institute of Technology (The Technion) at Haifa for permission to use a set of lecture notes prepared by their staff following a course I gave at their invitation in the spring of 1972. I am indebted to the University of Sussex for allowing me to include some examination questions among the exercises.

Finally, I wish to record my appreciation of the courtesy and patience which the Cambridge University Press has shown me during the preparation of this book.

W.L.

July, 1976