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0521337429 - Integral Equations: A Practical Treatment, from Spectral Theory to Applications

David Porter and David S. G. Stirling

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