
Notation Index

Symbol	Description	page
\bar{A}	closure of A	356
A_m	m th trace	223
$B(X), B(X, Y)$	spaces of operators	352
$B(x, y)$	beta function	364
$C([a, b]), C^\infty([a, b])$	spaces of functions	360
$\det A$	determinant of A	209
E^\perp	orthogonal complement of E	356
\mathcal{F}	field of scalars (\mathbb{R} or \mathbb{C})	95
I	identity operator	55, 352
$J(p)$	particular functional	248
$J_K(p)$	particular functional	204
$J(p, q)$	particular functional	260
$K^2, K^{1/2}$	power of operator K	10, 167
$\ K\ $	norm of operator K	352
$L_1(S)$	space of Lebesgue integrable functions	359
$L_2(a, b)$	space of square-integrable functions	351, 359
$\mathcal{N}(T)$	nullspace of T	103, 356
$O(h^n)$	order	205
$R_K(\phi)$	Rayleigh quotient	141
$\mathcal{R}(T)$	image (range) of T	103, 356
$\operatorname{sgn}(x)$	sign of x	–
T^*	adjoint of T	356
\bar{z}	complex conjugate	–
$\Gamma(x)$	gamma function	364
λ, μ	parameters	4, 96
$\mu_n, \mu_n(K)$	eigenvalues	141
μ_n^+, μ_n^-	eigenvalues	141
$\rho(T)$	spectral radius of T	86
$\sigma(T)$	spectrum of T	96

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$\phi_n, \phi_n^+, \phi_n^-$	eigenvectors	141
$\phi_y(x)$	particular function	308
$\psi(x)$	digamma function	365
$(a \leq x \leq b)$	for all x in $[a, b]$	2
$\ \cdot\ $	norm	351, 352
(ϕ, ψ)	inner product (also ordered pair)	352

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