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Integral equations

A practical treatment, from spectral theory to applications

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Preface

It often happens that the most concise and illuminating method of solving even the most practical problem in mathematics involves the use of abstract ideas and techniques. This is particularly true of integral equations, where much progress can be made by using both direct and abstract techniques side by side.

The advantage of reformulating an equation, such as an integral equation, as an 'abstract' problem in a Hilbert space is that many of the important issues become clearer. In the abstract setting, a function is regarded as a 'point' in some suitable space and an integral operator as a transformation of one 'point' into another. Since a point is conceptually simpler than a function this view has the merit of removing some of the mathematical clutter from the problem, making it possible to see the salient issues more clearly. It is thus easy to visualise elegant general structures which can be translated into results about the original concrete problem. To obtain these results in a useful form, however, a second step is needed, for elegant general results tend to produce only elegant generalities and a further process is required to recover hard specific facts about the solutions sought. We use the abstract framework of functional analysis to derive the general structures and more *ad hoc* techniques for the recovery.

There is all too often a gap between the approaches of a pure and an applied mathematician to the same problem, to the extent that they may have little in common. We offer, in this book, a middle road where we develop, rigorously, the general structures associated with the problems which arise in application areas and also pay attention to the recovery of information of practical interest. We do not, however, pursue the structural results to the ultimate generality where this would not have any bearing on the issues which motivate our study, integral equations. Conversely, we do not avoid substantial matters of calculation where these are necessary to adapt the general methods to cope with classes of integral equations which arise in application areas. Our approach is, we hope, both rigorous and satisfying to the pure mathematician and accessible and useful to those dealing with concrete problems.

The book deals with linear integral equations, that is, equations involving an unknown function which appears under an integral sign (and where the dependence on this function is linear). Such equations occur widely in diverse areas of applied mathematics and physics. They offer a powerful (sometimes the only) technique for solving a variety of practical problems. One reason for this utility is that all of the conditions specifying an initial value or boundary value problem for a differential equation can often be condensed into a single integral equation. In the case of partial differential equations the dimension of the problem is reduced in this process so that, for example, a boundary value problem for a partial differential equation in two independent variables transforms into an integral equation involving an unknown function of only one variable. This reduction of what may represent a complicated mathematical model of a physical situation into a single equation is itself a significant step, but there are other advantages to be gained by replacing differentiation with integration. Some of these advantages arise because integration is a 'smoothing' process, a feature which has significant implications when approximate solutions are sought. Whether one is looking for an exact solution to a given problem or having to settle for an approximation to it, an integral equation formulation can often prove to be a useful way forward. For this reason integral equations have attracted attention for most of this century and their theory is well-developed.

This book has its origins in a course of lectures given for a number of years at the University of Reading to final year Honours Mathematics and M.Sc. students. One of the aims of this course has been to bring together strands from pure mathematics and applied mathematics, often regarded by students as totally unconnected. To do this we have, of necessity, had to remain within areas that one student might be expected to know, so that we have not presumed, for example, a familiarity with specific applications, but have taken the mathematical modelling of these as given. For similar reasons we have not presumed knowledge of more sophisticated analysis, such as distribution theory, even if there are places where a little extra progress might have resulted.

We have followed the same guidelines in the book, which is also aimed at final year undergraduate and first year postgraduate students. We assume that the reader is familiar with classical real analysis, basic linear algebra and the rudiments of ordinary differential equation theory. In addition, some acquaintance with functional analysis and Hilbert spaces is necessary, roughly at the level of a first course in the subject, although we have found that even a limited familiarity with these topics is easily consolidated as a by-product of using them in the setting of integral equations. Because of the scope of the text and the emphasis on practical issues, we hope that the book will prove useful to those working in application areas who find that they need to know about, or more about, integral equations. Functional analysts, too, may find it useful in providing concrete examples of their art.

Although the treatment is our own, our ideas have of course been influenced by others. We have both felt for many years that integral equations should be treated in the fashion of this book and we derived much benefit from reading H. Hochstadt's text *Integral Equations*, Wiley–Interscience, 1973, and F. Smithies' now classic *Integral Equations*, Cambridge University Press, 1958. Other influences, in some cases acting more in spirit by making us aware of the sort of results we might seek, have been M. J. Sewell's *Maximum and Minimum Principles*, Cambridge University Press, 1987, papers by J. B. Reade (for certain eigenvalue approximations), by A. S. Peters (for the concept of a simplifying operator) and by I. H. Sloan (for the iterated Galerkin method). Most of the material in the book has been known for many years, although not necessarily in the form in which we have presented it, but the later chapters do contain some results we believe to be new.

Finally we should like to thank Rosemary Pellew for her skill, patience and enthusiasm in typing the manuscript at the same time as coping with the mysteries of a new word processor.

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