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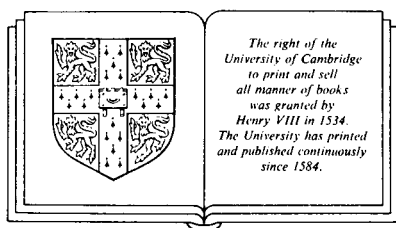
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# *Rings and factorization*

David Sharpe

*Department of Pure Mathematics  
University of Sheffield*



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*To my parents*

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## PREFACE

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This little book is about rings and factorization, but it is intended to be an exhaustive study of neither of these. If such a study exists, it will have to be sought in the standard works on the subject. The present aim is more modest. The book arose out of a short series (sequence?) of 20 lectures given to second year mathematics students at the University of Sheffield. As any university teacher will testify, by no means all mathematics students feel at home with abstract ideas, let alone see the point of them. The aim of the course was to help students to make the transition into a more abstract world as painlessly as possible by presenting abstract ideas in a fairly concrete context.

I am grateful to my students for suffering the inadequacies of my exposition. It is my hope that many if not all of them gained some pleasure as well as knowledge from the course, and that readers of this book will do the same.

Exercises have been included in most sections, with an indication of how to solve most of them at the end. I appeal to readers not to cheat by looking at the answers before they have had a go themselves; this is as bad as reading a whodunnit by starting at the last page!

It is assumed that readers are familiar with the basic ideas of sets and mappings, including the notions of injective (one-one), surjective (onto) and bijective (one-one and onto) mappings, although they do not appear prominently in the book; they are principally in Section 1.4. It is also assumed that readers are familiar with the idea of an equivalence relation on a set, and of how it partitions a set into subsets. These topics will be found in a textbook on elementary set theory.