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978-0-521-33717-5 - An Introduction to Hilbert Space
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University Printing House, Cambridge CB2 8BS, United Kingdom

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www.cambridge.org
Information on this title: www.cambridge.org/9780521337175

First published 1988
14th printing 2011

A catalogue record for this publication is available from the British Library

Library of Congress Cataloguing in Publication data

Young, Nicholas
An introduction to Hilbert space.

Includes index.
1. Hilbert space. I. Title.
QA322.4.Y68 1988 515.7′33 87-24903

ISBN 978-0-521-33071-8 Hardback
ISBN 978-0-521-33717-5 Paperback

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FOREWORD

The basic notions of the theory of Hilbert space are current in many parts of pure and applied mathematics, and in physics, engineering and statistics. They are well worth a place in any honours mathematics course, and Chapters 1 to 8 of this book aim to present them in a way accessible to undergraduate students. A course in Hilbert space is likely to be the last analysis course for many students, and it should therefore be able to stand on its own: it should not depend for its motivation on further study of abstract analysis, but should as far as possible have a value which is apparent either on aesthetic grounds or for its scientific or practical applications. For this reason I have included more historical and background material than is customary, and have omitted some of the major theorems about Banach spaces which are traditionally taught in introductory courses on functional analysis, but which are really more appropriate to students who will be pursuing operator theory further (the closed graph, Hahn–Banach and uniform boundedness theorems). The second half of the book describes two substantial applications. One of these is standard: the Sturm–Liouville theory of eigenfunction expansions, and its role in the solution of the partial differential equations of mathematical physics by the method of separation of variables. The other (in Chapters 12 to 16) is less common, but is nevertheless ideal for a final year course. It is beautiful mathematics, it is relatively recent and visibly useful. It also entails the development of some standard operator theory along the way, and exhibits very well the connection between abstract analysis and the more classical field of complex analysis.

Although the book was written primarily for an undergraduate audience, I hope it may be found useful for graduate courses also. I firmly believe that functional analysis is best approached through a sound grounding in Hilbert space theory, and am confident that students will be better able to benefit from one of the many excellent advanced texts on functional analysis and its applications if they first master the material contained herein. Chapters 12 to 16 may also be of interest to some electrical engineers. Some recent developments, particularly in control and filter design, require familiarity with this aspect of operator theory.

Chapters 1 to 8 are based on a compulsory course of twenty lectures which I gave to third year honours students at Glasgow University, and the remainder of the book, with a few omissions, on an optional twenty lecture course for fourth year students. In forty lectures at the undergraduate level it should be possible to cover the whole book except for the Adamyan–Arov–Krein theorem and the proofs of Fatou's theorem and the existence of square roots of positive operators. Chapters 12 to 16 do not depend on Chapters 9 to 11: they can be read straight after Chapter 8.

The book presupposes introductory courses in real analysis, linear algebra and topology (metric spaces suffice). For Chapters 12 to 16, and some of the problems earlier in the book, elementary complex analysis is required. It is tacitly assumed in Chapters 9 to 11 that the reader has met differential equations before, though formal requirements are slight. I have taken pains *not* to assume knowledge of the Lebesgue integral: the reader is asked only to believe that there is a definition of integral which makes $L^2(a, b)$ complete and the continuous functions a dense subspace. However, I am obliged to admit that there are parts of Chapter 13 which will feel distinctly more comfortable to those who are familiar with Lebesgue measure.

I am grateful to Dr Frances Goldman and Dr Philip Spain for reading the text and making useful suggestions. I am also very thankful that, despite the all-conquering march of the word processor, Cambridge University Press was willing to accept manuscript, so that I do not have to thank anyone for his excellent typing.