

1 Temperature

1.1 INTRODUCTION

Thermal physics is the study of those properties of materials which are affected by temperature. It is an enormous field, having something to say about subjects as diverse as the expansion of a solid, the internal constitution of stars and why the electrical resistance of some metals vanishes at low temperatures.

The job of the scientist is to observe nature, to try to recognize the regularities in its behaviour and to seek to link those regularities together by laws and principles. For example, it is often found that the current flowing in an electrical conductor is proportional to the applied potential difference; this regularity is called Ohm's law.

When the scientist tries to 'explain' his observations, there are two levels of explanation he can attempt. He may content himself with ideas relating to the behaviour of matter in bulk (e.g. Ohm's law). This is the *macroscopic* approach. In the area of thermal physics it leads to the subject called *thermodynamics*. This branch of physics developed most rapidly during the last century in connection with the study of machines, such as the steam engine, which supplied power for the new industries; but, it was soon realized that the laws of thermodynamics were very fundamental and of importance in areas quite different from power engineering where they were developed. The laws of thermodynamics provide a theoretical framework which is used in many branches of modern science. A typical result which may be derived by thermodynamics is the Clausius–Clapeyron equation (page 111) which connects the variation with temperature of vapour pressure with latent heat.

At the other level of explanation, the scientist tries to base his understanding on ideas of the nature and properties of matter at the atomic level. This is *microscopic* physics. An example is the kinetic theory of gases (chapter 3) which 'explains' the bulk behaviour of gases in terms of the properties of the mole-

cules of which the gas is composed. While models and laws relating to matter at the atomic level play no direct part in macroscopic physics, they are the essence of microscopic physics. In thermal physics, the microscopic approach leads to the subject called *statistical mechanics*. A typical result of the arguments of statistical mechanics is the Maxwell distribution (page 54), which gives the probabilities of different molecular speeds in a gas.

Both macroscopic and microscopic approaches are of value, as we shall see later in this book, for there are times when we need to explain in terms of fundamental laws operating at the atomic level, while there are other times when to involve ourselves with detailed microscopic models would be a positive encumbrance.

1.2 SOME BASIC IDEAS

We call the object we are investigating the *system*. It could be a volume of gas, or a spring, or a refrigerator, or a solid in the process of melting. We shall often illustrate our arguments by taking as a model system a given mass of gas in a cylinder with a frictionless piston. This is a good model to take because it is easy to visualize what happens when we heat it or do work on it by compressing it.

We describe the *state* of the system in terms of appropriate *parameters* or *variables*, such as mass, pressure, volume, density, temperature. When a system is in a given state, we will always get the same results for any measurements we may make on it. The variables are not all independent; some are related to one another. For example, density is mass/volume. For simple systems of given mass we find that we need to fix the values of *two* variables in order to fix the state of the system. For our given mass of gas, for example, we find that, if we first set the volume, we may still adjust the pressure to any value we please (by varying the temperature), but once the pressure is set also, there is no other property which can be varied. We therefore

say that the gas has two *degrees of freedom*: we are free to choose two of the variables as we like, but then all the other parameters will have taken up definite values which we cannot adjust. This means that any mathematical equation connecting system variables must have at least three variables in it: the values of two must be known in order to determine the state of the system and so fix the value of a third. Thus, the equation of state of an ideal gas (page 40) connects pressure, volume and temperature: $pV/T = \text{constant}$. Again, this is why we have two common heat capacities. A heat capacity is the rate at which heat is absorbed as we change the temperature (section 2.3); but since simple systems have two degrees of freedom, we do not know *how* the system is going to change as the temperature is changed unless more information is given. Two simple cases are that the system should be kept at constant volume or at constant pressure. Such a condition is called a *constraint* because it constrains the system to change in a certain way. Any constraint removes the second degree of freedom so that now a definite amount of heat is absorbed as the temperature is changed.

In developing the ideas of temperature, heat, and so on, we shall be interested in the ways systems interact with one another. There are two kinds of interaction. In *work-like* interactions, one system does work on another (figure 1.1): a force moves through a given distance as in compressing a gas or stretching a spring, or a battery will do work in charging a capacitor because it forces charge to flow against an opposing potential difference. The other kind of interaction is *thermal* and is typified by flow of heat when we place a hot body in contact with a cold one (figure 1.2). In both kinds of interaction,

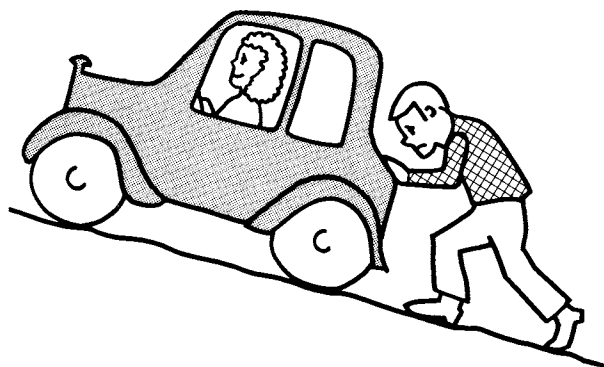


Figure 1.1 A work-like interaction

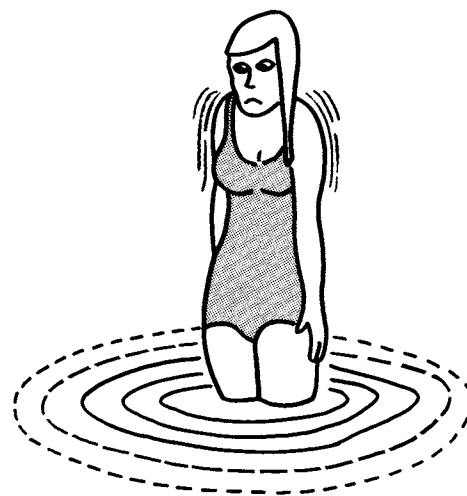


Figure 1.2 A thermal interaction

energy is transferred from one system to another; the difference is that in work-like interactions the process involves some sort of large scale motion (all the molecules of the piston move forward together as the gas is compressed), whereas in thermal interactions the energy is associated with disordered thermal motions of the atoms of which the system is composed: the random motions of the molecules of a gas or the vibrations of the atoms of a solid. When a thermal interaction takes place, the atoms of the cooler system are excited into more violent thermal motion by contact with the hotter (more energetic) atoms of the hotter system.

If two systems are placed in contact in such a way as to allow thermal interaction, they are said to be in *thermal contact* (figure 1.3). If a system is prevented from interacting thermally with its surroundings it is said to be *thermally isolated* (coffee in a vacuum flask), and any change it undergoes is said to be an *adiabatic change*. Put differently, an adiabatic change is one in which no heat enters or leaves the system. We shall discuss the ideas of heat and work more fully in chapter 2.

If you have found these introductory ideas confusing at this stage, it is probably because I have been giving a kind of preview of the things we shall be discussing carefully later in the book. Perhaps you should return to sections 1.1 and 1.2 when you have finished reading it!

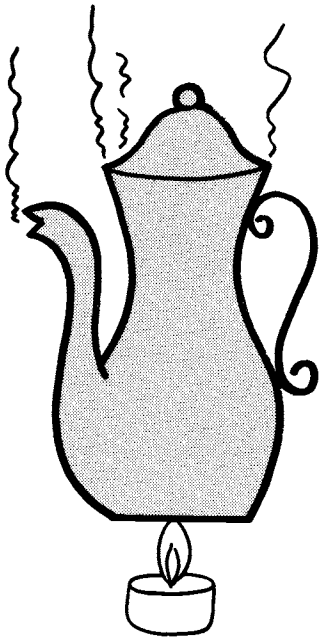


Figure 1.3 Two systems making thermal contact

1.3 TEMPERATURE

The idea of temperature almost certainly originally arose from the physiological sensation of hotness, an unreliable measure of temperature (figure 1.4). We can, however, develop a more exact concept of what temperature means by discussing what happens when bodies are placed in thermal contact.

If we take two systems and place them in thermal contact we generally find that initially changes will occur in both (figure 1.5). Eventually the changes cease, and the systems are then said to be in *thermal equilibrium*. We introduce the idea of temperature by saying that *the condition for the systems to be in thermal equilibrium is that they should be at the same temperature*. Conversely, two systems which are already at the

same temperature will not undergo change when placed in thermal contact. Equality of temperature is therefore the condition for thermal equilibrium.

However, defined like this, it is not clear that the condition for thermal equilibrium might not depend on the nature of the systems concerned. Can the same condition apply when we put a thermometer in our mouth to take our temperature as applies when we put a thermometer in a beaker of concentrated sulphuric acid? Experience shows that the condition for thermal equilibrium does not depend on the nature of the systems concerned. This experimental fact is embodied in the *zeroth law of thermodynamics* (so called because the need for it was not recognized until after the first law had been established).

If two systems are separately in thermal equilibrium with a third, then they must also be in thermal equilibrium with each other.

Since we are free to choose anything we like for the third system, it follows that the condition for thermal equilibrium cannot depend on the nature of the systems concerned: all systems in thermal equilibrium have the same temperature irrespective of their nature. Another way of expressing this is to say that temperature is a universal property. The kind of experiment with which we might illustrate the zeroth law is shown in figure 1.6.

We should note that if we require one system to be in thermal equilibrium with another (i.e. at the same temperature) this represents a constraint on the system and removes one of its degrees of freedom. With our fixed mass of gas, for example, in the absence of any constraint, we are free to choose pressure and volume as we please; however, as soon as we require thermal equilibrium with some other system, the temperature is fixed, and for every value of volume there is only one possible value of pressure. We can put this another way by saying that for each temperature there is a

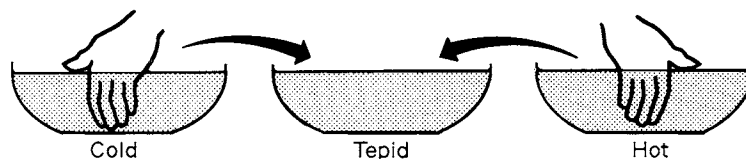


Figure 1.4 The physiological sensation of hotness is an unreliable measure of temperature. After one's hand has been in cold water for a time, tepid water feels hot. After it has been in hot water, the tepid water feels cold.

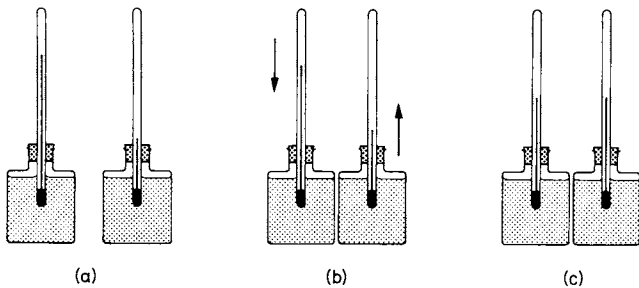


Figure 1.5 When two systems (a), are placed in thermal contact (b), changes generally take place in both systems until they reach thermal equilibrium (c). Here, thermometers indicate the changes (of temperature).

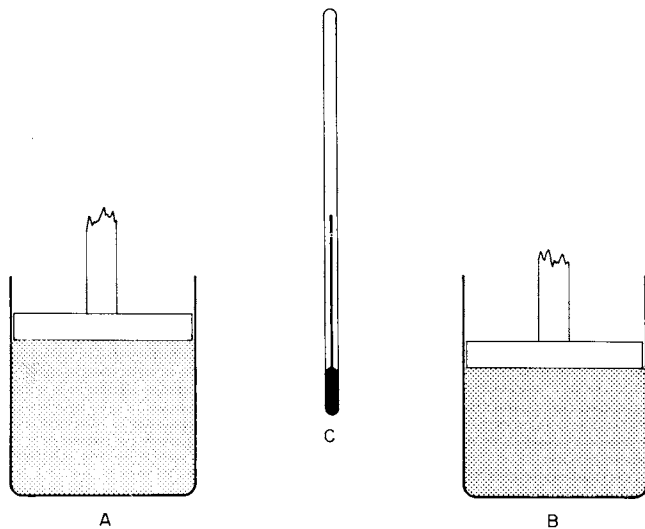


Figure 1.6 An illustration of the zeroth law. Systems A and B are fixed masses of gases at particular values of pressure and volume. The third system, C, is a thermometer. If C is in thermal equilibrium separately with A and B, so that its reading does not change when it is placed in thermal contact with either, then no changes will take place when A and B are placed in thermal contact with each other (because they are already at the same temperature).

unique relationship between pressure and volume. This is represented mathematically by an equation of the form

$$f(p, V) = \Theta$$

where $f(p, V)$ stands for the formula which connects the pressure and volume to the temperature Θ .*

* The Greek letter *theta*. For temperature we use the capital letter. The lower case letter, θ , is generally used for angles.

For an ideal gas, f is a simple product, pV , and for constant temperature we have

$$f(p, V) = pV = \Theta = \text{constant}$$

which is Boyle's law (page 40). The curve relating p to V for a given value of Θ is called an *isotherm* (figure 1.7).

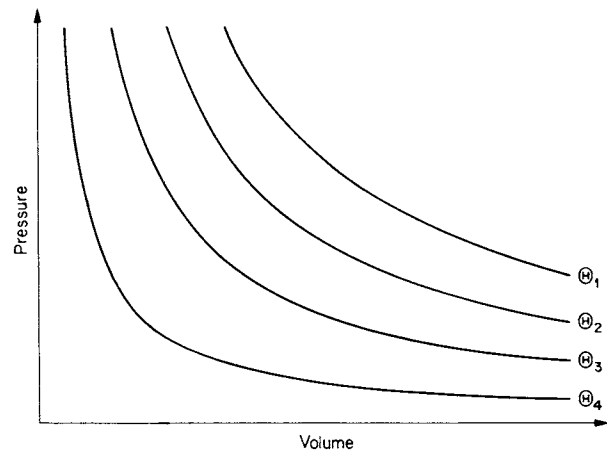


Figure 1.7 Isotherms for an ideal gas. For each temperature, there is only one possible value of pressure for each value of volume.

1.4 SCALES OF TEMPERATURE

We have introduced temperature as a rather abstract concept connected with thermal equilibrium. In practice, we would like to be able to represent temperature by a number whose magnitude changes in some regular way in relation to our ideas of hotness: the hotter the body, the larger the number representing temperature. This is what we do when we set up a *scale of temperature*. The easiest way to do this is to choose a convenient system with a property x which changes with temperature and take the value for temperature as linearly proportional to x :

$$\Theta(x) = ax + b \tag{1.1}$$

where a and b are constants. Here, again, the convention written $\Theta(x)$ reminds us that Θ is a quantity whose value depends on the value of x : Θ is said to be a *function* of x . The relationship of equation 1.1 is illustrated in figure 1.8.

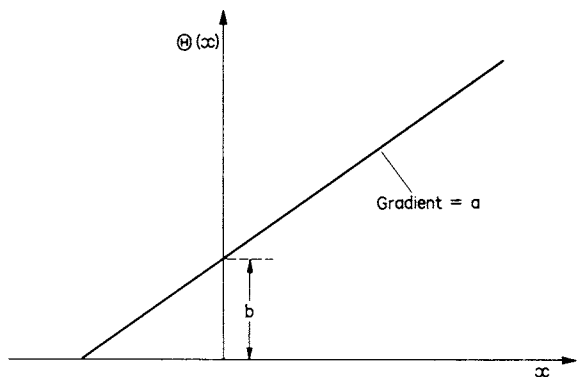


Figure 1.8 A plot of the equation $\Theta(x) = ax + b$

When we set up a temperature scale for everyday use, we want a conveniently placed zero, say low in the range of commonly encountered temperatures, and a sensibly sized unit. Conventionally, the *ice point*, the temperature at which water freezes at one atmosphere pressure, is taken as 0° , and the *steam point*, the temperature at which water boils at one atmosphere pressure, is taken as 100° . A scale so constructed is known as a *centigrade* scale. (Centigrade means one hundred steps.) We may take some examples.

A temperature on a centigrade scale based on the expansion of mercury in a mercury in glass thermometer uses the length of mercury in the capillary as the *thermometric* (temperature measuring) quantity x . The centigrade temperature is given by

$$\Theta(L) = (L - L_i) \times \frac{100}{(L_s - L_i)} \quad (1.2)$$

where L , L_s and L_i are the lengths of the mercury at the temperature to be measured and at the ice and steam points respectively (figure 1.9). Comparing

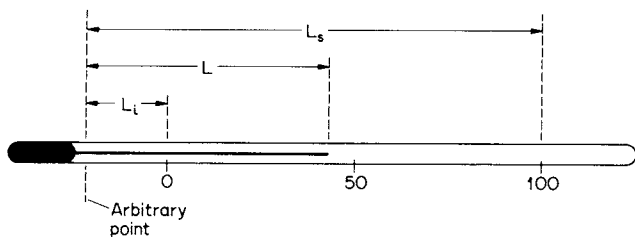


Figure 1.9 Constructing a centigrade scale for a mercury-in-glass thermometer

equation 1.2 with the general linear form, equation 1.1, we find

$$a = 100/(L_s - L_i)$$

$$b = -100L_i/(L_s - L_i)$$

Again, a temperature on a centigrade scale based on the variation of the resistance R of a coil of copper wire is given by

$$\Theta(R) = 100(R - R_i)/(R_s - R_i) \quad (1.3)$$

where R , R_i and R_s are the values of the resistance at the temperature to be measured, and at the ice and steam points respectively.

Scales constructed in this way are known as *empirical* scales: empirical means based on experiment. The symbol Θ is always used for empirical temperature.

When the scales are set up like this, we find that the value obtained for a temperature depends on what thermometer we use. This is because different properties do not respond in the same way to change of temperature. As a result, centigrade scales based on different systems will not generally agree with one another except, of course, at 0° and 100° , the calibration points, where they must agree by definition. Figure 1.10 shows the differences between centigrade temperatures determined with different thermometers.

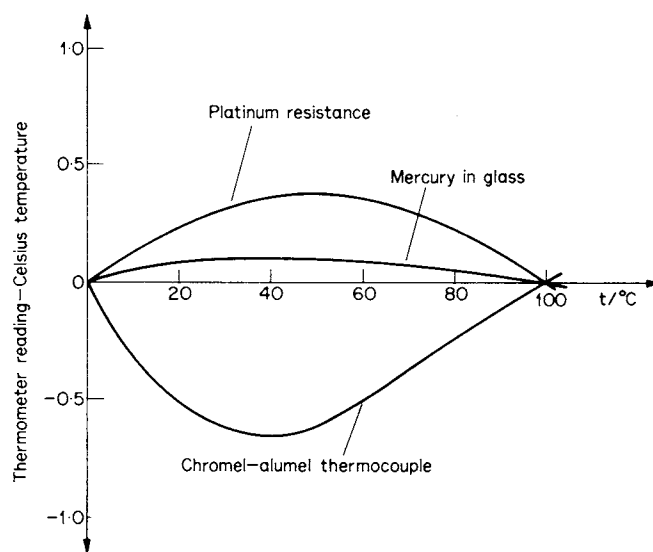


Figure 1.10 Differences between centigrade scales of common thermometers over the temperature range $0 - 100^\circ\text{C}$

In the search for a scale of temperature which did not depend on the properties of particular substances it was found that disagreement was small among thermometers based on the behaviour of gases. Gases have two degrees of freedom, so that a constraint has to be applied if the pressure or volume is to be uniquely related to temperature. Commonly, volume is kept constant and pressure used as the thermometric property. This is a *constant volume gas thermometer* (page 8) and will give a centigrade temperature

$$\Theta(p) = 100(p - p_i)/(p_s - p_i) \quad (V = \text{constant})$$

Now while disagreements are generally small when temperatures are determined with different gases at normal pressures in this way, it is found that the differences which are present become smaller as the pressures used are reduced. If the measurements are *extrapolated* to find the temperature which would be given if the pressure could be reduced to zero (figure 1.11), it is found that, in this limit, *all* gas thermometers

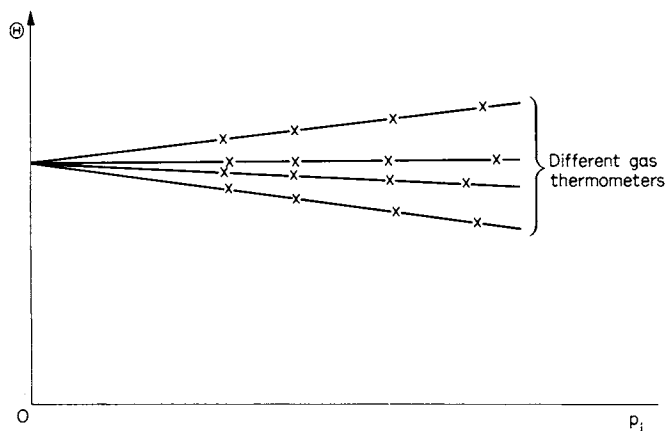


Figure 1.11 All gas thermometers give the same value for a temperature in the low pressure limit. With each thermometer, the temperature is found using different values of p_i . If the lines through the experimental values of temperature are extended back to find the temperature which would be given if the measurements could be made with $p_i = 0$, it is found that all the thermometers give the same value for the temperature.

give the *same* value for a temperature. One thus obtains a temperature scale which is independent of the properties of any particular substance. This scale is called the *perfect gas centigrade scale*. On the perfect gas centigrade scale, the expression for the temperature

is written

$$\Theta_{pgc} = \lim_{p_i \rightarrow 0} 100(p - p_i)/(p_s - p_i) \quad (V = \text{constant}) \quad (1.3)$$

$\lim_{p_i \rightarrow 0}$ means that we take the number given by the formula when the measurements are extrapolated to the limit where $p_i = 0$ (figure 1.11).

If measurements made on this scale are extrapolated back to find the perfect gas centigrade temperature at which the pressure of the perfect gas would vanish (figure 1.12), the value found is -273.15 . This temperature is known as *absolute zero*.

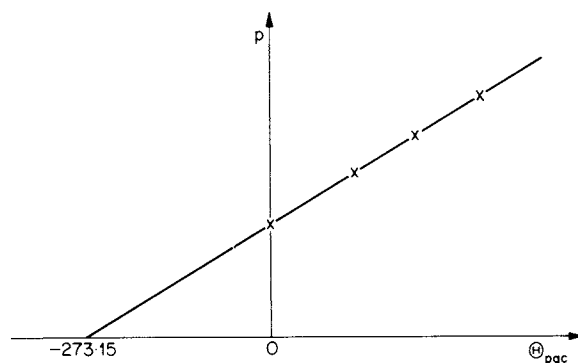


Figure 1.12 The pressure of an ideal gas would vanish at a perfect gas centigrade temperature of -273.15

1.5 THERMODYNAMIC TEMPERATURE

The reason why all gases give the same value for a temperature in the low pressure limit is that, in that limit, their behaviour tends to that of the perfect or ideal gas (chapter 3) whose equation of state is

$$pV/T = \text{constant} \quad (1.4)$$

where T is *thermodynamic temperature*, the fundamental measure of temperature which comes from the second law of thermodynamics (chapter 7). For the moment the equation must be taken on trust. (When we discuss the ideal gas in chapter 3, we shall show that its equation of state is $pV/\Theta = \text{constant}$, where Θ is temperature on the perfect gas scale. It is not until chapter 7, when we discuss the second law, that we are able to explain the idea of thermodynamic temperature and prove that perfect gas temperature

is identical to thermodynamic temperature.) Substituting 1.4 in 1.3, the volume and the constant cancel through numerator and denominator and we get

$$\Theta_{pgc} = (T - T_i) \times \frac{100}{(T_s - T_i)}. \quad (1.5)$$

This equation shows that perfect gas centigrade temperatures, like thermodynamic temperatures, are independent of the properties of any particular substance.

Thermodynamic temperature is now accepted as the fundamental measure of temperature. Gases are unique in that thermodynamic temperature appears in such a simple way in the equation of state (which holds for real gases in the low pressure limit), and this is why determinations of thermodynamic temperatures are often ultimately based on gas thermometry.

Now we are free to choose the size of the unit of thermodynamic temperature as we please. If we choose to have 100 units between ice and steam points, equation 1.5 becomes

$$\Theta_{pgc} = (T - T_i)/K \quad (1.6)$$

However, there are disadvantages in fixing the size of the unit in this way. In the first place it is necessary to calibrate a gas thermometer at *two* fixed points: the ice and steam points. Secondly, when measurements made at the fixed points are extrapolated back to very low temperatures, that is to small values of T , the experimental uncertainties become *relatively* large, which could be serious in low temperature work. Now we notice that thermodynamic temperature has a natural zero, namely, the temperature at which the pressure of an ideal gas would vanish (see equation 1.4). If we take this natural zero as a fixed point on the scale, we only need to calibrate the thermometer at *one* fixed point to fix the size of the unit. In effect, we choose the *value* of thermodynamic temperature for *one* fixed point and this sets the scale. This way of fixing the size of the unit of thermodynamic temperature is the one now adopted by the International Committee of Weights and Measures. The fixed point chosen is the *triple point of water* (the temperature at which water, ice and water vapour coexist in equilibrium, a temperature more reproducible than the ice or steam points) and the value of temperature allotted to it is 273.16. The unit so

defined is called the kelvin* for which the symbol K is used. Thus

Thermodynamic temperature is the fundamental temperature; its unit is the kelvin which is defined as the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

Therefore, a thermodynamic temperature determined by gas thermometry would be given by

$$T/K = \lim_{p \rightarrow 0} (pV) \times \frac{273.16}{\lim_{p \rightarrow 0} (pV)_{tr}} \quad (1.7)$$

where pV is the value of the product of pressure and volume at the temperature to be determined and $(pV)_{tr}$ is the value of the product at the triple point. Absolute zero is 0 K by definition. When we discuss the ideal gas we shall see that absolute zero is the temperature at which all thermal motion would cease.

Figure 1.13 shows schematically how the triple point is achieved for thermometer calibration.

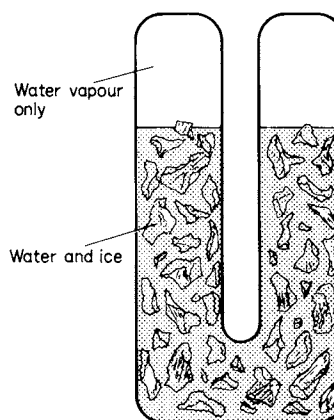


Figure 1.13 A triple point cell. The cell is cooled until some ice is present as well as water and water vapour. When all three are in equilibrium with one another, the temperature is 273.16 K by definition. The thermometer to be calibrated is inserted into the central tube.

1.6 THE CELSIUS TEMPERATURE SCALE

The reason for the choice of 273.16 K for the thermodynamic temperature of the triple point of water

* After Lord Kelvin (William Thompson), 1824–1907.

was that it gave *to the accuracy of the best measurements then available* exactly 100 K between ice and steam points. This meant that, within experimental error, the perfect gas centigrade scale and thermodynamic temperature coincided precisely apart from the shift of zero to the ice point:

$$\theta_{pgc} = T/K - 273.15 \quad (1.8)$$

This is the same as 1.6 but with the value of T_i substituted. However, more recent measurements have revealed previously unsuspected errors in conventional gas thermometry. In particular, it has been shown that, to a precision better than 0.01 K, $T_s - T_i = 99.97$ K. We therefore now have:

$$\left. \begin{array}{l} \text{triple point} = 273.16 \text{ K} \text{ by definition} \\ \text{ice point} = 273.15 \text{ K} \\ \text{steam point} = 373.12 \text{ K} \end{array} \right\} \text{by experiment}$$

Equation 1.5 still defines a perfect gas *centigrade* scale, of course, because the term $100/(T_s - T_i)$ takes account of the improved value for T_i ; but the centigrade unit so defined is now known to be slightly smaller than the kelvin. For most purposes, the difference is unimportant, but in precision work there could be confusion. Anticipating this possibility, the International Committee of Weights and Measures defined a *new* scale, the *Celsius scale*, by the equation

$$t/^{\circ}\text{C} = T/K - 273.15 \quad (1.9)$$

where t is the temperature in degrees Celsius. The Celsius degree is therefore identical to the kelvin by *definition*, and Celsius and thermodynamic temperatures differ only by the shift of zero. Note that Celsius is not a new name for centigrade: there will always be 100 centigrade units between ice and steam points because that is how a centigrade scale is defined, but experiment shows the temperature differences to be significantly less than 100 kelvins.

The temperatures of the primary reference points given in table 1.1 are the values adopted to define the International Practical Temperature Scale of 1968 (see section 1.8). In the next few years, a new set of temperatures will be allotted to the reference points and a new practical scale will be defined. The maximum correction will be less than 0.5 K.

Note that the symbol t is always used for Celsius temperatures and, whereas no degree sign is used with K for the kelvin, the degree sign is included with the C for Celsius temperatures: $^{\circ}\text{C}$.

1.7 SOME COMMON THERMOMETERS

In choosing a thermometer for a particular application the important criteria are

- accuracy*, the ability to give an accurate value for the thermodynamic temperature,
- sensitivity*, the ability to measure very small changes of temperature,
- suitability*, satisfying special requirements like small size, rapid response, robustness, resistance to corrosion, ability to operate remote from the observer,
- convenience*, simplicity of construction and operation.

The best choice in any particular case depends on the task in hand.

Gas thermometers are important because they have been widely used for basic determinations of thermodynamic temperature; but they are inconvenient, and, in order to obtain a good degree of accuracy, elaborate precautions have to be taken and complicated corrections made. The essential elements of a simple constant volume gas thermometer are shown in figure 1.14. The bulb B is immersed in the region whose temperature is to be measured. The tube connecting the bulb to the manometer is usually capillary so that the amount of gas not at the temperature being measured is relatively small. For the same reason, the bulb is usually relatively large (in

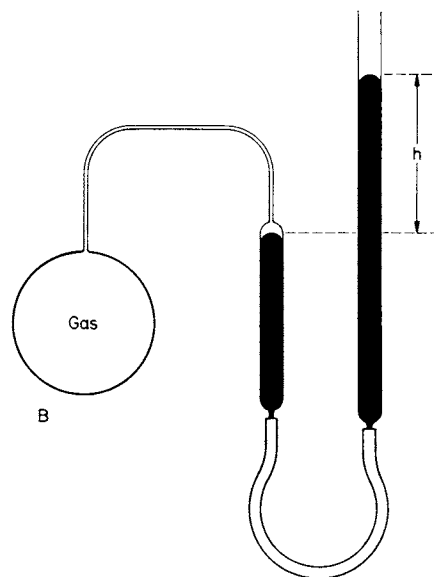


Figure 1.14 The elements of a simple constant volume gas thermometer

Table 1.1 Temperatures of the standard and primary reference points as adopted for the International Practical Temperature Scale of 1968. Temperatures are given to greater precision than the experimental uncertainties because they are used as calibration points of the International Practical Temperature Scale (see section 1.8).

	T/K	$t/^\circ\text{C}$	Estimated uncertainty ³ /K
<i>Standard</i>			
Triple point of water ¹	273.16	0.01	exact by definition
<i>Primary</i>			
Triple point of hydrogen ²	13.81	-259.34	0.01
Boiling point of hydrogen ² at 25/76 atm pressure	17.042	-256.108	0.01
Boiling point of hydrogen ² at 1 atm pressure	20.28	-252.87	0.01
Boiling point of neon at 1 atm pressure	27.102	-246.048	0.01
Triple point of oxygen	54.361	-218.789	0.01
Boiling point of oxygen at 1 atm pressure	90.188	-182.962	0.01
Boiling point of water at 1 atm pressure	373.15	100.00	0.005
Melting point of zinc at 1 atm pressure	692.73	419.58	0.03
Melting point of silver at 1 atm pressure	1235.08	961.93	0.2
Melting point of gold at 1 atm pressure	1337.58	1064.43	0.2

¹ The water should have the isotopic composition of ocean water.

² There are two types of molecular hydrogen, *ortho* and *para*. The reference points require the two types to be in their equilibrium proportions at the temperatures concerned.

³ But see section 1.6 for the implications of more recent measurements.

accurate work perhaps 10^{-3} m^3). The volume is kept constant by adjusting the height of the right hand arm of the manometer so that the level of the mercury in the left stays constant. Corrections have to be made for the 'dead space' (the volume of gas not at the temperature being measured), changes of atmospheric pressure, variation of the density of mercury with temperature, expansion of the bulb, and so on. By using a gas which only liquefies at low temperatures (helium or hydrogen, for example) deviations from the ideal gas law are negligible unless the thermometer is being used for low temperature measurements, in which case the pressure must be kept low.

When the various corrections may be neglected, a constant volume gas thermometer may conveniently be calibrated at the ice point and the thermodynamic temperature found using the formula

$$T/\text{K} = 273.15 p/p_i \quad (1.10)$$

Because they are inconvenient, gas thermometers are rarely used and, in order to be able to calibrate other thermometers accurately, the thermodynamic temperatures of several reference points have been determined with great precision using gas thermometry. Table 1.1 lists the temperatures of the standard and primary reference points as adopted in 1968.

Thermometers based on the *expansion of liquids* are very convenient for many purposes and, although they are not particularly accurate, they can be made very sensitive. Over a moderate range of temperature their scale is nearly linear in thermodynamic temperature (i.e. expansion is directly proportional to increase in thermodynamic temperature). Commonly used liquids are mercury and ethyl alcohol, which cover the ranges -39 to $+350^\circ\text{C}$ and -117 to $+78^\circ\text{C}$ respectively.

Thermometers based on the *expansion of solids* are relatively crude, but convenient for some applications such as the activation of thermostats or for driving the pens of thermographs. Usually, thin strips of dissimilar metals are bonded together to form a *bimetallic strip* which flexes as the temperature is changed, because the metal on one side expands more than the metal on the other (figure 1.15).

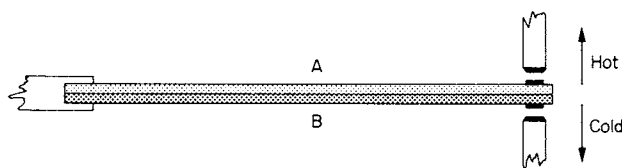


Fig. 1.15 A simple thermostat. If metal *B* has a greater expansivity than *A*, the bimetallic strip will flex upwards if the temperature rises and connection will be made with the upper contact.

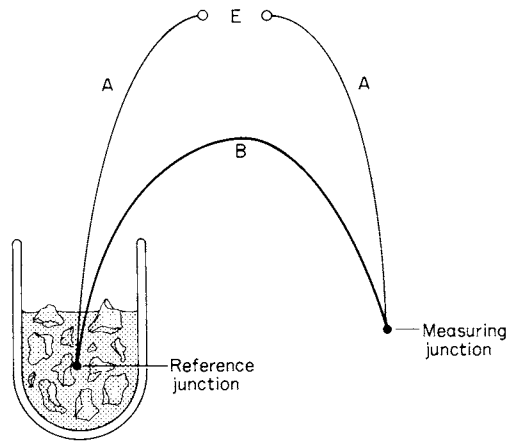


Figure 1.16 A thermocouple circuit using metals *A* and *B*. One junction is held at a fixed reference temperature (at the ice point, for example) and the other is at the temperature to be measured. If the thermoelectric effects in the two metals are unequal, there is a net e.m.f. *E* around the circuit when the junctions are at different temperatures.

Thermocouples make use of the variation with temperature of the Seebeck e.m.f., the e.m.f. developed in a circuit of dissimilar conductors when the junctions are at different temperatures (figure 1.16). The e.m.f. arises because increasing the temperature of the conductor increases the thermal motions of the electrons which, in turn, makes them want to expand towards a cooler region. The extent to which they tend to move away from the hot regions depends on the conductor concerned so that, with a circuit of dissimilar conductors, the effects do not cancel one another and there is a net e.m.f. around the circuit. If one junction is kept at a constant temperature, the e.m.f., *E*, developed depends on the temperature *t* of the other junction in a non-linear way (i.e. it is not directly proportional); but the relationship can usually be represented reasonably accurately over a moderate range of temperature by a few terms of a power series:

$$E = a_0 + a_1t + a_2t^2 + a_3t^3 + \dots \quad (1.11)$$

in which the constants have to be determined by calibration against reference points. For small temperature changes, the higher power terms can often be neglected. The e.m.f.'s are usually rather small and are also rather sensitive to impurities in the conductors used, so that much accuracy is difficult to achieve, but thermocouples are often very convenient: they cover a large temperature range, they may be made very small so as to be able to explore regions where the

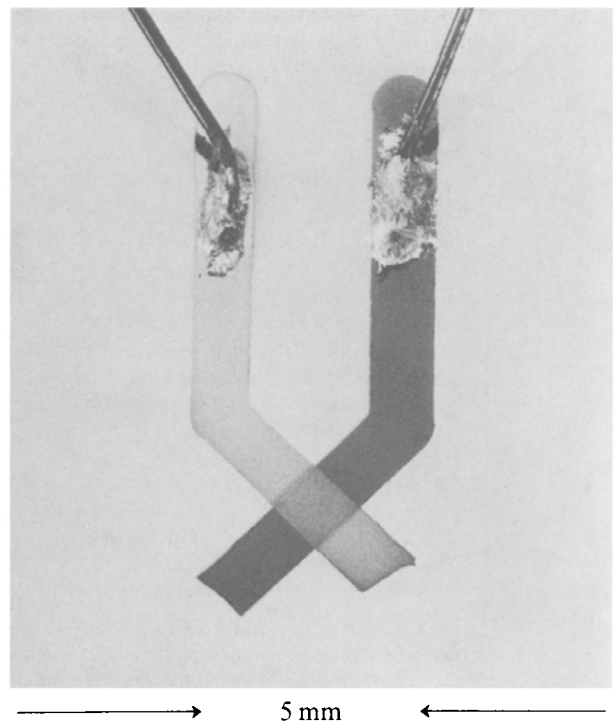


Figure 1.17 A thermocouple of thin films of copper and lead deposited on glass to measure the surface temperature. Connection to the films is made with thin wires of the same metals, soldered on with indium.

temperature is varying (figure 1.17), and they can respond rapidly. They are also useful for remote sensing (e.g. measuring the temperature in a furnace), since long connecting wires do not affect their operation. The approximate sensitivities (*a*₁ in equation 1.11) of several commonly used thermocouples are listed in table 1.2.

Table 1.2 Approximate sensitivities of common thermocouples

Thermocouple	Sensitivity/ $\mu V K^{-1}$
copper—constantan ^a	40
iron—constantan ^a	50
chromel ^b —alumel ^c	40
Pt—Pt + 10% Rh	6

^a also called Eureka: 60% Cu, 40% Ni.

^b 90% Ni, 10% Cr.

^c 95% Ni plus Al, Si, Mn.

Resistance thermometry traditionally uses the variation with temperature of the electrical resistance of metals. The resistance increases with increasing temperature because the thermal motions (vibrations)