FOREWORD

In 1951 the Mathematics Faculty in Cambridge asked J. E. Littlewood to give a talk, of about forty minutes, at the first of a series of 'social evenings'. A little later, the 'Archimedeans' — the Cambridge undergraduate mathematical society — invited him to address them. These two talks are the origins of a collection of essays for the general public which were published in 1953 as *A Mathematician's Miscellany*.

The volume was a great success. 'This admirable book is impossible to summarise. It overflows with what G. B. Shaw calls "the gaiety of genius" ' wrote one reviewer. 'For many of us this is the book of the year' added another. Several letters addressed to Littlewood began: 'What a delightful book.' From the many reactions to the book, Littlewood concluded that 'The loftier the intellect, the more the appreciation. The dim deprecate it.'

The *Miscellany* was reprinted several times but for the past twenty or so years it has been out of print. After writing the volume, Littlewood lived for another quarter of a century and went on collecting material for a new expanded edition: curiosities, howlers, strange anecdotes and various recollections of life at Trinity College. Much of that material is incorporated in this edition, together with the essay 'The Mathematician's Art of Work' which Littlewood wrote in 1967 and based on his collection of anecdotes.

The *Miscellany* remains a marvellous and attractive piece of work. However, the enjoyment and understanding of the reader will certainly be enhanced by knowing something of Littlewood's life and the environment at Trinity College in Cambridge, which formed the setting for so many of his stories. As in the original edition, a section marked by * is likely to be too technical for a non-mathematician.

The mathematical life in England in the first half of this century was dominated by two giants, Hardy and Littlewood. In the 1920s Edmund Landau, the eminent German number theorist, expressed the view that 'The mathematician Hardy-Littlewood was the best in the world, with Littlewood the more original genius and Hardy the better journalist.' To have deserved such praise is an extraordinary achievement at the best of times, but to appreciate it properly, we must recall that just a few years earlier England had no analysts to speak of. In the nineteenth century, France and Germany could boast of many outstanding pure mathematicians, and England, especially Cambridge, did have excellent applied mathematicians, including Green, Stokes, Adams, Lord Kelvin, Airy and Maxwell. However, in pure mathematics England produced only a handful of algebraists, among them Cayley and Sylvester, and failed to produce any notable analysts. This sorry state of affairs was changed by Littlewood and Hardy: by 1930 the school of analysis established by them was second to none.

John Edensor Littlewood was born at Rochester on 9 June 1885, the eldest of three children of Sylvia Maud Ackland and Edward Thornton Littlewood. His mother was part Irish, but his father was of pure British folk: farmers and landowners. Thorntons from the Eastern Counties, Robinsons of Suffolk, Stotherts of Scotland, Kitcheners of Binsteal and Littlewoods of Baildon Hall, Bradford. The father of the eminent journalist and diarist Henry Crabb Robinson was an ancestor of John Edensor Littlewood, and so was the great-grandfather of Lord Kitchener. It is recorded that a member of the family of Littlewood fought at Agincourt, and many branches of the family tree can be traced to the sixteenth century. This is not to say that Littlewood himself cared about his family tree: a mathematical proof containing gaps reminded him of being descended from William the Conqueror — with two gaps.

In recent centuries Littlewood's ancestors had been farmers, landowners, ministers, schoolmasters, printers, publishers, editors and doctors. Although Cambridge, according to Littlewood, inspired an awe equalled to nothing felt since, both his father and paternal grandfather were Cambridge men. The Reverend William Edensor Littlewood (1831–86) was educated at Pembroke College and was bracketed 35th wrangler in the mathematical tripos. This was the grandfather whose middle name, given in honour of his grandmother, Sarah Edensor, from the village of Edensor in Derbyshire, was passed on to John Edensor Littlewood. The eldest son of the Reverend Littlewood, Edward Thornton Littlewood (1859–1941), went to Peterhouse and was ninth wrangler. At the time College Fellowships were awarded on the basis of Tripos results and, but for a misplaced 'old school tie' attitude, he would have been Fellow of Magdalene. His College, Peterhouse, had no Fellowship to offer and he refused to take his parson father's advice and apply for one at

Magdalene, which went to a lower Wrangler. In his old age Littlewood remarked with a certain amount of sadness that his childhood would have been very different had his father stayed in Cambridge.

As it happened, Edward Littlewood accepted the headmastership of a newly founded school at Wynberg, near Cape Town, and took his family there in 1892. J. E. Littlewood spent eight years of his childhood in South Africa, the beauty of which made an impression on him that he never forgot. Having grown out of the schools there, he went to the Cape University, but his father realised that his mathematical education would suffer if he stayed in South Africa, so in 1900 he was sent to St. Paul's School in London. He spent almost three years at the school, under the guidance of F. S. Macaulay, an unusually able mathematician, who in 1928 became a Fellow of the Royal Society. (Not surprisingly, it was Littlewood who proposed him.) Littlewood's work at the school and his subsequent life at Cambridge are admirably related in *A Mathematical Education*, so I will not dwell on the details.

Littlewood took the Entrance Scholarship Examination of December 1902 and although he was expected to do well, he found the papers too difficult and got only a minor scholarship at Trinity College.

On arriving in Cambridge, Littlewood began work for the *Mathematical Tripos*. In its prime the Mathematical Tripos was far and away the most severe mathematical test that the world has ever known, one to which no university today (including Cambridge) can show any parallel. The Examination evolved during the eighteenth century; from 1753 on the candidates were divided into three classes: Wranglers, Senior Optimes and Optimes. In order to establish a *strict order of merit*, the examination was turned into a high-speed marathon: four days of tests of up to ten hours a day. This absurd examination produced few excellent pure mathematicians, but it was phenomenally successful in training outstanding applied mathematicians.

The candidates in the Tripos worked under various *coaches*, who drilled their men mercilessly. In spite of the name, a good coach was usually a respectable mathematician and, occasionally, a very good one. Littlewood was lucky to have the last of the great coaches, R. A. Herman, who was a contemporary and a friend of his father, and a Fellow of Trinity. To be in the running for Senior Wrangler, the top man in the order of merit, undergraduates had to spend two-thirds of the time practising how to solve difficult problems against time, and this is what Littlewood did.

In Littlewood's time Part I of the Mathematical Tripos was a three-year course but occasionally scholars used to take Part I at the end of their second year. Littlewood did this in 1905, while still 19, and was bracketed Senior Wrangler with Mercer, who had graduated from Manchester University before coming to Cambridge.

The Senior Wranglers were celebrities in Cambridge and their photos were sold during May Week. When a friend of his tried to buy one of him, he was told: 'I'm afraid we're sold out of Mr Littlewood but we have plenty of Mr Mercer.'

Littlewood as Senior Wrangler.

A few years later, in 1910, Hardy played a decisive role in abolishing the strict order of merit in the Tripos, and was a sworn enemy of the milder examination which replaced it. Littlewood was also firmly against the order of merit; he thought that his first two years at Cambridge were wasted although he did not feel that the system caused him any real harm. As Hardy wrote later: 'He understood that the mathematics he was studying was not the real thing and regarded himself as playing a game. It was not exactly the game he would have chosen, but it was the game which the regulations prescribed, and it seemed to him that, if you were going to play the game at all, you might as well accept the situation and play it with all your force. He believed that he could play the game as well as any of his rivals, and he was right.' He even felt a satisfaction of a sort in successful craftsmanship.

Littlewood took Part II of the Mathematical Tripos in his third year. Although he was learning genuine mathematics he wasted a good deal of time in the ordinary course of trial and error. At the end of the third year, in the Long Vacation of 1906, Littlewood began research under E. W. Barnes (later Bishop of Birmingham). As the first project, Barnes suggested study of entire functions of order zero. After a few months Littlewood's efforts resulted in a fifty page paper.

Encouraged by Littlewood's success, Barnes suggested another problem: 'Prove the Riemann Hypothesis.'

The *Riemann Hypothesis* (R.H.) is, by general concensus, the most important unsolved problem in Pure Mathematics. *The zeta function of Riemann is defined for $s = \sigma + it$, σ and t real, $\sigma > 1$, by

$$\zeta(s) = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \dots$$

This function is regular in the half-plane $\sigma \geq 1$ and it has an analytic continuation throughout the s plane, having a simple pole at $s = 1$. At first sight $\zeta(s)$ is a peculiarly defined complex function but, in fact, it is intimately related to the distribution of primes. Indeed, the great eighteenth century mathematician Leonhard Euler knew that

$$\zeta(s) = \prod_p \left(1 + \frac{1}{p^s} + \frac{1}{p^{2s}} + \frac{1}{p^{3s}} + \dots\right)$$

where the product is taken over all primes.

The main questions about $\zeta(s)$ concern the distribution of its zeros. It is known that every negative even integer is a zero of $\zeta(s)$ (these are the *trivial zeros*) and that infinitely many zeros lie in the *critical strip* $0 < \sigma < 1$. In 1860, the outstanding German mathematician Riemann conjectured that all non-trivial zeros lie on the *critical line* $\sigma = 1/2$. This is the Riemann Hypothesis, which is still open today. In terms of the distribution of prime numbers, R.H. means that the primes are fairly regularly distributed.

The *Prime Number Theorem* (P.N.T.), proved independently by Hadamard and de la Vallée Poussin, asserts that $\pi(x)$, the number of primes up to x, is about $x/\log x$, and the *logarithmic integral* $li(x) = \int_0^x \frac{dt}{\log t}$ is an even better approximation. (The integral is defined by its *Cauchy principal value*:

$$li(x) = \lim_{\epsilon \to 0} \left\{ \int_0^{1-\epsilon} + \int_{1+\epsilon}^{x} \right\}.)$$

Assuming R.H., the P.N.T. can be improved to

$$|\pi(x) - li(x)| \leq Cx^{1/2} \log x,$$

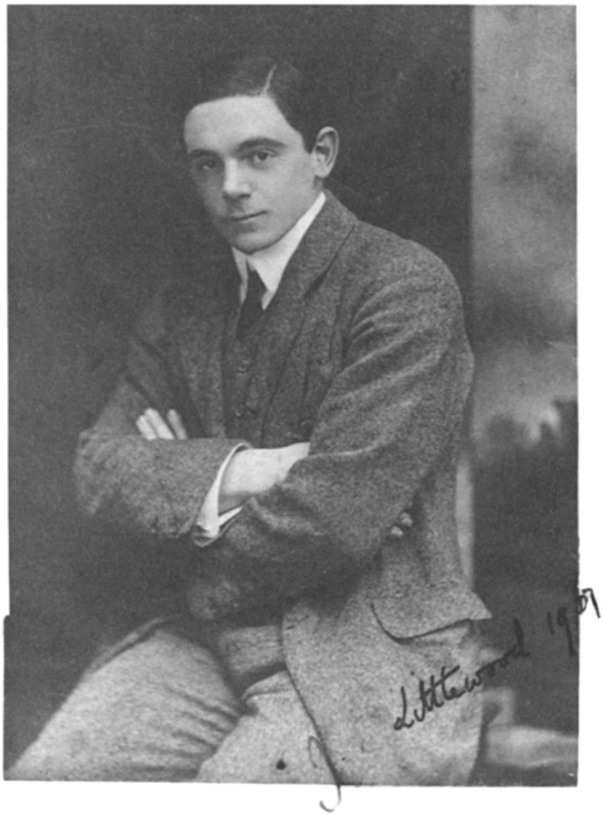
where C is some absolute constant. (In one of his most celebrated papers, Littlewood later proved that, contrary to all the numerical evidence, the difference $\pi(x) - li(x)$ changes sign infinitely often, (see page 100)*

Barnes did not know that R.H. was connected to the distribution of primes, although that had been proved on the continent several years before, and Littlewood had to discover it for himself: assuming R.H., he deduced the Prime Number Theorem. This was just in time for his first Fellowship dissertation.

Trinity offers annually a number of junior fellowships to its graduates, who have three opportunities of competing: at the end of the fourth, fifth and sixth years following their matriculation. Littlewood competed at his first opportunity, in September 1907. The dissertation was well received and would have secured his election if there had not been a candidate in classics competing at his last chance whom the electors considered to deserve election. Littlewood was informed that his election in the following year, 1908, was a certainty, and the election duly took place. The first paper which made Littlewood famous, published in 1912, was also about some consequences of the Riemann hypothesis.

Meanwhile Littlewood had been offered the Richardson lectureship in the University of Manchester. Though at £250 this was better than the usual £150 or £120, he did not gain financially, but felt he needed a change from Cambridge. On looking back he considered that it was a disaster on his part to accept it, for he was greatly overworked during the three years of his tenure. He always spoke of it as his period of *exile*. Once, during his period of exile, he walked along a river in Manchester, and it looked like ink. Presently a tributary ran into it, making an inky trace on the surface. King John sprang to his mind: 'Hell darkened as he entered it.'

Littlewood joined the Trinity staff in 1910, replacing Whitehead. This coincided with new mathematical interests. Landau's fundamental book on analytical number theory had been published only a year earlier,



Littlewood in 1907.

which enabled Hardy and Littlewood to catch up with the latest results in number theory and to confront the analytical problems they give rise to. In 1920 Littlewood succeeded Hardy to the Cayley lectureship in the University.

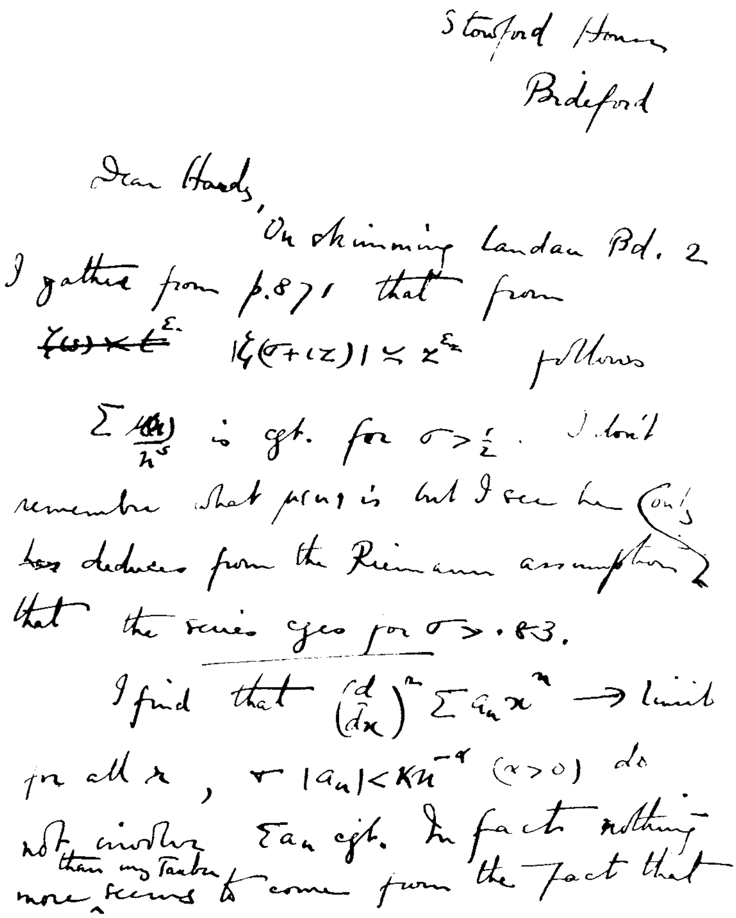
In 1912 he moved into a large set of rooms on the first floor of Nevile's Court. He occupied these for the next 65 years, until his death, except during the First World War, when he served as Second Lieutenant in the Royal Garrison Artillery. Throughout all these years, he was a much loved and respected Fellow of Trinity. He felt perfectly at home in the College and was deeply attached to it. He never cared for College office but nonetheless he played a key role in the society and, for several decades, shaped its development by serving as a Fellowship Elector.

Shortly before the War, Hardy and Littlewood began their extraordinarily successful collaboration, lasting for 35 years — surely the most successful collaboration ever in mathematics! They wrote a hundred joint papers, with their last publication being published a year after Hardy's death. In addition, with Pólya, they wrote an excellent book entitled *Inequalities*, published by CUP in 1934, which is widely used to this day.

There are many reasons why the Hardy-Littlewood collaboration flourished. They had a number of common interests, especially summability, inequalities, Diophantine approximation and its connections to function theory, Fourier series and the theory of numbers, inspired by Landau's *Primzahlen*. They were both geniuses, completely dedicated to mathematics. Hardy was, perhaps, more stylish, a man of intellectual panache, interested in beautiful patterns, but Littlewood was imaginative and amazingly powerful, enjoying the challenge of a very difficult problem.

This period is vividly described by the eminent Danish mathematician Harald Bohr, at a lecture given on his sixtieth birthday in 1947 (*Collected Works*, vol. I, pp. *xxvii* – *xxviii*, 1953, Dansk Mat. Forening, reproduced with permission).

> Already early in life, I had the good fortune to come into close professional contact — which later turned into an intimate friendship — with the two only slightly older English mathematicians, Hardy and Littlewood, who were to bring English pure mathematics to such a high standard. Thus I often had occasion to take a trip to Cambridge, the classical centre of English mathematics and natural sciences since the days of Newton, and the old university town of Oxford, with which Hardy was connected for some years. Life in the old English university colleges — for me it was Trinity College in Cambridge and New College in Oxford — could not but captivate and enchant everyone. While everything was permeated and marked by venerable traditions, unbroken through centuries, at the same time there reigned a rare spirit of freedom and tolerance, and not only was it allowed, but it was even appreciated that even the most individual and divergent opinions were expressed undisguisedly, often in extreme form, though never in an offensive manner.
>
> To illustrate to what extent Hardy and Littlewood in the course of the years came to be considered as the leaders of recent English mathematical research, I may report what an excellent colleague once jokingly said: 'Nowadays, there are only three really great English mathematicians: Hardy, Littlewood and



Stowford House
Bideford

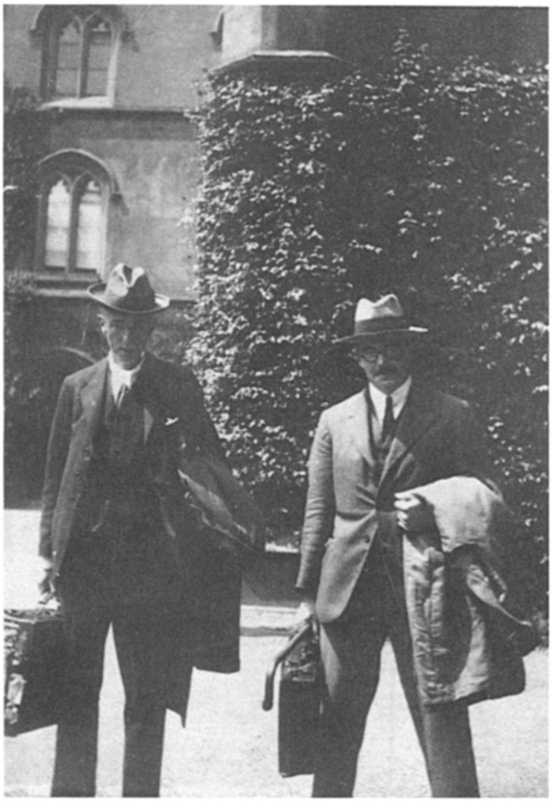
Dear Hardy,
On skimming Landau Bd. 2 I gather from p. 871 that from ~~$\zeta(s) \prec t^{\epsilon}$~~ $|\zeta(\sigma + iz)| \prec z^{\epsilon}$ follows

$\sum \frac{\mu(n)}{n^s}$ is cgt. for $\sigma > \frac{1}{2}$. I don't remember what proof is but I see he (only) deduces from the Riemann assumption that the series cges for $\sigma > \cdot 83$.

I find that $\left(\frac{d}{dx}\right)^n \sum a_n x^n \to$ limit for all x, & $|a_n| < K n^{-\alpha}$ $(\alpha > 0)$ do not involve $\sum a_n$ cgt. In fact nothing more than my Tauber seems to come from the fact that

A letter from Littlewood to Hardy, *c.*1910

Hardy-Littlewood.' The last refers to the marvellous collaboration through the years between these two equally outstanding scientists with their very different personalities. This cooperation was to lead to such great results and to the creation of entirely new methods, not least in the theory of numbers, that to the uninitiated, they almost seemed to have fused into one. To illustrate the strong feelings of independence which, as a part of the old traditions, are so characteristic of the English spirit, I should like to tell how Hardy and Littlewood, when they planned and began their far-reaching and intensive team work, still had some misgivings about it because they feared



Hardy and Littlewood in New Court, Trinity College.

that it might encroach on their personal freedom, so vitally important to them. Therefore, as a safety measure, (it was, as usual when they worked out something together, Hardy who did the writing), they amused themselves by formulating some so-called 'axioms' for their mutual collaboration. There were in all four such axioms. The first of them said that, when one wrote to the other (they often preferred to exchange thoughts in writing instead of orally), it was completely indifferent whether what they wrote was right or wrong. As Hardy put it, otherwise they could not write completely as they pleased, but would have to feel a certain responsibility thereby. The second axiom was to the effect that, when one received a letter from the other, he was under no obligation whatsoever to read it, let alone to answer it, — because, as they said, it might be that the recipient