

CONTENTS

Introduction	1
Historical survey	3
Algorithms for describing $\text{Wh}(G)$	4
Survey of computations	14
Part I General theory	20
Chapter 1. Basic algebraic background	21
1a. Orders in semisimple algebras	21
1b. p -adic completion	27
1c. Semilocal rings and the Jacobson radical	32
1d. Bimodule-induced homomorphisms and Morita equivalence	37
Chapter 2. Structure theorems for K_1 of orders	40
2a. Applications of the reduced norm	40
2b. Logarithmic and exponential maps in p -adic orders	50
Chapter 3. Continuous K_2 and localization sequences	63
3a. Steinberg symbols in $K_2(\mathbb{R})$	64
3b. Continuous K_2 of p -adic orders and algebras	70
3c. Localization sequences for torsion in Whitehead groups	73
Chapter 4. The congruence subgroup problem	90
4a. Symbols in K_2 of p -adic fields	91
4b. Continuous K_2 of simple $\hat{\mathbb{Q}}_p$ -algebras	100
4c. The calculation of $C(\mathbb{Q}[G])$	115
Chapter 5 First applications of the congruence subgroup problem	127
5a. Constructing and detecting elements in SK_1 : an example	127
5b. $Cl_1(\mathbb{R}[G])$ and the complex representation ring	134
5c. The standard involution on Whitehead groups	148
Chapter 6. The integral p -adic logarithm	153
6a. The integral logarithm for p -adic group rings	153
6b. Variants of the integral logarithm	166
6c. Logarithms defined on $K_2^C(\hat{\mathbb{Z}}_p[G])$	168
Part II Group rings of p -groups	172
Chapter 7. The torsion subgroup of Whitehead groups	173
Chapter 8. The p -adic quotient of $SK_1(\mathbb{Z}[G])$: p -groups	183
8a. Detection of elements	183
8b. Establishing upper bounds	191
8c. Examples	200

Chapter 9. $Cl_1(\mathbb{Z}[G])$ for p -groups	205
Chapter 10. The torsion free part of $Wh(G)$	229
Part III General finite groups	243
Chapter 11. A quick survey of induction theory	245
11a. Induction properties for Mackey functors and Green modules	246
11b. Splitting p -local Mackey functors	254
Chapter 12. The p -adic quotient of $SK_1(\mathbb{Z}[G])$: finite groups	272
Chapter 13. $Cl_1(\mathbb{Z}[G])$ for finite groups	291
13a. Reduction to p -elementary groups	291
13b. Reduction to p -groups	305
13c. Splitting the inclusion $Cl_1(\mathbb{Z}[G]) \subseteq SK_1(\mathbb{Z}[G])$	322
Chapter 14. Examples	328
References	340
Index	348