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WHITEHEAD GROUPS OF FINITE GROUPS

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PREFACE

This book is written with the intention of making more easily accessible techniques for studying Whitehead groups of finite groups, as well as a variety of related topics such as induction theory and p-adic logarithms. It developed out of a realization that most of the recent work in the field is scattered over a large number of papers, making it very difficult even for experts already working with K- and L-theory of finite groups to find and use them. The book is aimed, not only at such experts, but also at nonspecialists who either need some specific application involving Whitehead groups, or who just want to get an overview of the current state of knowledge in the subject. It is especially with the latter group in mind that the lengthy introduction — as well as the separate introductions to Parts I, II, and III — have been written. They are designed to give a quick orientation to the contents of the book, and in particular to the techniques available for describing Whitehead groups.

I would like to thank several people, in particular Jim Davis, Erkki Laitinen, Jim Schafer, Terry Wall, and Chuck Weibel, for all of their helpful suggestions during the preparation of the book. Also, my many thanks to Ioan James for encouraging me to write the book, and for arranging its publication.

LIST OF NOTATION

The following is a list of some of the notation used throughout the book. In many cases, these are defined again where used.

Groups:

$N_G(H)$, $C_G(H)$ denote the normalizer and centralizer of H in G

$G^{ab} = G/[G,G]$ (the abelianization) for any group G

$S_p(G)$ denotes a p-Sylow subgroup of G

C_n denotes a (multiplicative) cyclic group of order n

$D(2n)$, $Q(2n)$, $SD(2n)$ denote the dihedral, quaternion, and semidihedral groups of order $2n$

S_n, A_n denote the symmetric and alternating groups on n letters

$H \rtimes G$ denotes a semidirect product where H is normal

$G \wr C_n$ and $G \wr S_n$ denote the wreath products $G^n \rtimes C_n$ and $G^n \rtimes S_n$

$M^G = \{x \in M\colon Gx = x\} \cong H^0(G;M)$ and $M_G = M/\langle gx-x\colon g \in G,\ x \in M\rangle \cong H_0(G;M)$ if G acts linearly on M (groups of invariants and coinvariants)

Fields and rings:

$\hat{K}_p = \hat{\mathbb{Q}}_p \otimes_{\mathbb{Q}} K$ if K is any number field and p a rational prime (so $\hat{K}_p$ is possibly a product of fields)

$\hat{R}_p = \hat{\mathbb{Z}}_p \otimes_{\mathbb{Z}} R$ if R is the ring of integers in a number field

μ_K, $(\mu_K)_p$ (K any field) denote the groups of roots of unity, and p-th power roots of unity, in K

ζ_n $(n \geq 1)$ denotes a primitive n-th root of unity

ξ_n $(n \geq 0)$, when some prime p is fixed, denotes the root of unity $\exp(2\pi i/p^n) \in \mathbb{C}$.

$K\zeta_n$ (for any field K and any $n \geq 1$) denotes the smallest field extension of K containing ζ_n

$J(R)$ denotes the Jacobson radical of the ring R

$\langle - \rangle$ means "subgroup (or $\hat{\mathbb{Z}}_p$-module) generated by"

$\langle - \rangle_R$ means "R-ideal or R-module generated by"

$e^r_{ij} = e_{ij}(r)$ (where $i,j \geq 1$, $i \neq j$, and $r \in R$) denote the elementary matrix with single off-diagonal entry r in the (i,j)-position

<u>K-theory:</u>

$SK_1(\mathfrak{A}) = \mathrm{Ker}[K_1(\mathfrak{A}) \longrightarrow K_1(A)]$
$K_1'(\mathfrak{A}) = K_1(\mathfrak{A})/SK_1(\mathfrak{A})$
for any $\mathbb{Z}$- or $\hat{\mathbb{Z}}_p$-order $\mathfrak{A}$ in a semisimple $\mathbb{Q}$- or $\hat{\mathbb{Q}}_p$-algebra A

$Cl_1(\mathfrak{A}) = \mathrm{Ker}[SK_1(\mathfrak{A}) \twoheadrightarrow \oplus_p SK_1(\hat{\mathfrak{A}}_p)]$ for any $\mathbb{Z}$-order $\mathfrak{A}$

$C(A) = \varprojlim_I SK_1(\mathfrak{A}, I)$ for any semisimple $\mathbb{Q}$-algebra A and any $\mathbb{Z}$-order $\mathfrak{A} \subseteq A$, where the limit is taken over all ideals of finite index (see Definition 3.7)

$C_p(A)$ denotes the p-power torsion in the finite group $C(A)$

$Wh(R[G]) = K_1(R[G])/\langle rg\colon r \in \mu_K,\ g \in G\rangle$ and $Wh'(R[G]) = Wh(R[G])/SK_1(R[G])$ whenever R is the ring of integers in any finite extension K of $\mathbb{Q}$ or $\hat{\mathbb{Q}}_p$ (and G is any finite group)

$Wh'(G) = Wh(G)/SK_1(\mathbb{Z}[G]) = K_1'(\mathbb{Z}[G])/\langle \pm g\rangle$ for any finite group G

$K_2(R,I) = \mathrm{Ker}[K_2(R) \longrightarrow K_2(R/I)]$ for any ring R and any ideal $I \subseteq R$ (see remarks in Section 3a)

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