This book discusses the structure and dynamical properties of quantized vortex lines in superfluid helium-4 in the light of research on vortices in modern fluid mechanics. It is intended for researchers and graduate students in both fields, and it is the first book to give a comprehensive treatment of the problem.

Vortex motion is a huge subject which has permeated physics for centuries, and the author begins his discussion with a review of the knowledge of classical fluid dynamics which is relevant to the main topics of the book. This is followed by a presentation of basic material on helium II and quantized vortices, giving some historical background and establishing the language and principal phenomenology of the subject. The following chapters deal with different aspects of the subject including vortex dynamics and mutual friction, the structure of quantized vortices and their interaction with helium-3 atoms and ions, vortex arrays, and vortex waves, which are now a major preoccupation in superfluidity. The book concludes with substantial introductions to two currently active topics, namely superfluid turbulence and the nucleation of quantized vortices. Both quantum tunnelling and thermal activation nucleation processes are discussed, including the Kosterlitz–Thouless transition in thin films.

The comprehensive approach provided by the author will make this an invaluable book for students taking advanced undergraduate or graduate courses, and for all those involved in research on classical and quantum vortices.

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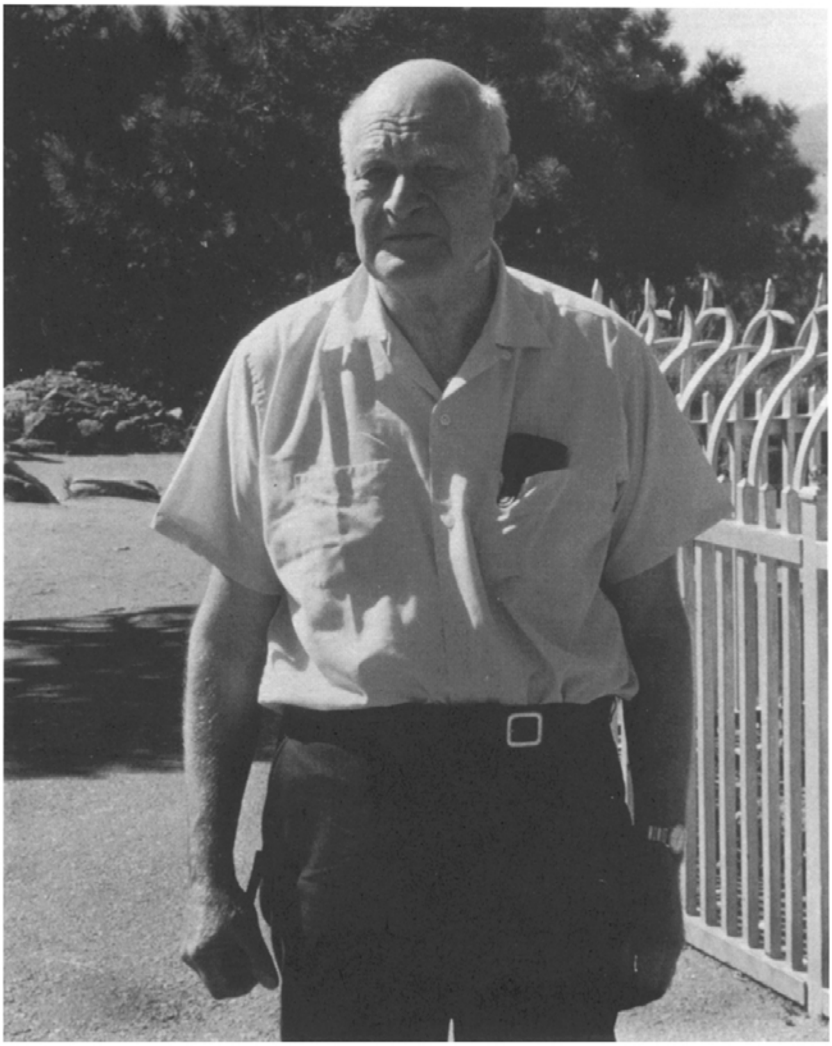
Quantized vortices in helium II



Lars Onsager: 1903–76. Photo: Author.

Quantized vortices in helium II

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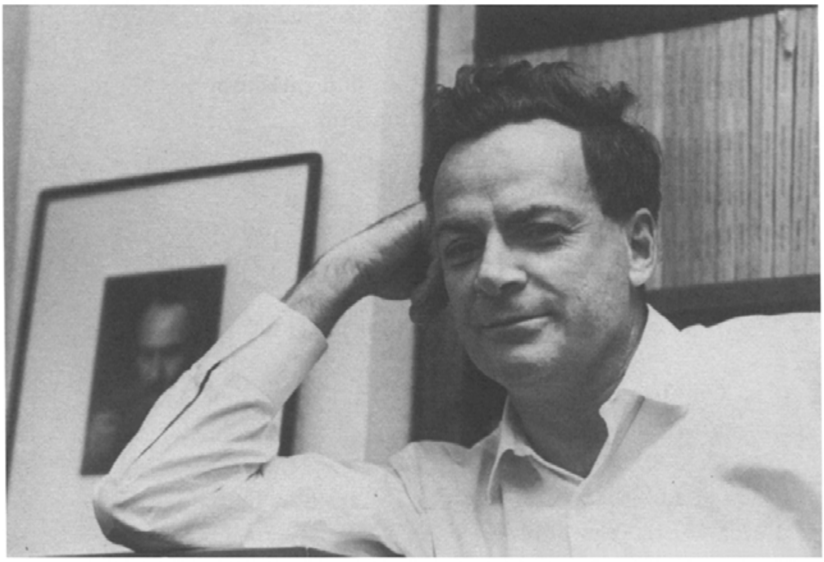
To the memory of
Lars Onsager and Richard P. Feynman

Contents



Richard P. Feynman: 1918–88. Photo: Archives of the California Institute of Technology.

Preface

Research on the properties of superfluid ^{4}He has fascinated physicists ever since the discovery of superfluidity in 1939 by Allen and Misener and Kapitza. For many years the two-fluid model and its ramifications were a vigorous area of research even though a microscopic theory of the fluid was not forthcoming. The subject has always been attractive to graduate students because of the fascinating combination of skills required for its understanding: quantum mechanics, statistical mechanics and hydrodynamics. With the passage of time and with the advent of new discoveries such as the superfluidity of ^{3}He the study of the two-fluid model and related properties has rightfully lost its place in center stage in low temperature physics, but in its place has emerged a valuable, and I think enduring, discipline: namely the study of specific issues where the unique properties of liquid helium offer a testing ground often unmatched elsewhere in nature. For example, we have the study of critical phenomena near the lambda transition of ^{4}He and its influence on the whole subject of phase transition experiments and theory. We have inelastic neutron scattering and its influence on such matters as the theory of elementary excitations and the question of Bose condensation. We have the study of quantized vortex line turbulence coming along as a new branch of turbulence research. Further examples are Bénard convection experiments in liquid helium and mixtures of ^{3}He in ^{4}He, and the development and testing of rigorous theories of the hydrodynamical properties of helium II, to mention only a subset of the wide range of topics being pursued today.

In the earlier days of liquid helium research there were several attempts to summarize all the known knowledge of the subject, beginning with Keesom's *Helium*. There followed Atkins', Wilks' and Keller's books each of which included extra topics such as mixtures of ^{3}He in ^{4}He and solid helium, and my own brief lecture notes at the University of Chicago written by William Glaberson and the late Peter Parks. The next major attempt to summarize the state of knowledge was that of Benneman and Ketterson, a two volume set with chapters by a group of many authors.

This book is a reflection of the tendency we have mentioned above, namely to consider modern research in ^{4}He physics as pertinent to the development of certain specialized topics. Our subject is vortices. Vortex motion is a huge subject which has permeated physics for centuries. We begin our discussion with a review of the knowledge of classical fluid dynamics relevant to the main topics of the book. Since so much of what we know about quantized vortices comes from classical fluid dynamics, we gain perspective by choosing this as a way to begin. On the other hand, the topics of Chapter 1 are a collection of disparate results which do not easily fit together as a single subject. Readers might well omit some of the specialized topics in Chapter 1 and refer back to them as needed to illustrate specific discussions of vortices in liquid helium. Chapter 2 contains basic material on helium II and quantized vortices, giving some historical background and establishing the language and principal phenomenology of the subject. Chapter 3 considers vortex dynamics and mutual friction, the force coupling the motion of the normal and superfluid components. Chapter 4 provides a discussion of the structure of quantized vortices and the interaction of ^{3}He and ions with vortices. Chapter 5 contains a discussion of distributions of vortices in helium II and various theories of their arrangements in steady flows. Vortex waves have become a major preoccupation in superfluidity which is discussed in Chapter 6. The book concludes with brief introductions to two specialized and currently active subjects, namely superfluid turbulence and the problem of the nucleation of quantized vortices. Thermal activation processes are discussed including the Kosterlitz–Thouless transition in thin films.

The purpose of this book is to acquaint the reader with modern research in vortices in liquid helium in the context of current vortex research in classical fluids. The background assumed is a knowledge of statistical and quantum mechanics at the beginning graduate level as well as some fluid mechanics.

Acknowledgements

Subjects in physics such as superfluidity progress because of the efforts of a very sizeable community of scholars.

Every one of us in indebted to the memory of Lars Onsager and Richard Feynman who almost overnight laid down the basic and enduring concepts of quantized vortices.

William F. Vinen and Henry E. Hall were the pioneering physicists who took the theoretical ideas of Onsager and Feynman designed experiments and interpreted the results in a way which has been of use to the field ever since. As Feynman once remarked to me: 'They made it real.'

I am personally indebted to my teachers and colleagues who have worked with me on vorticity over many years: Cecil T. Lane and Lars Onsager (my advisors), Joe Vinen, Subrahmanyan Chandrasekhar, Paul Roberts, Chris Jones, Ron Hills, Bill Glaberson, Klaus Schwarz, Carlo Barenghi and Tony Maxworthy. My graduate students who have worked on liquid helium with me include Howard Snyder, Brian Springett, David Tanner, Klaus Schwarz, Bill Glaberson, Peter Parks, Don Strayer, Michael Cromar, Bob Walden, Carlo Barenghi, Charles Swanson, Rabi Wang, John Pfotenhauer, Tom Wagner, Ben You, David Samuels, Paul Bunson and Greg Bauer. A number of persons mentioned were kind enough to review parts of the manuscript for me at various stages.

I am grateful to Bill Glaberson and North-Holland Publishing for permission to quote extensively from our joint article on vortices in *Progress in Low Temperature Physics*. This article formed the basis from which this book evolved. Many of the insights in that review and this book came from his ideas, research and graduate students.

I am forever indebted to Paul Roberts and Joe Vinen who have collaborated with my research group throughout my career and whose ideas and critical insight have been invaluable in preparing this account of quantized vortices.

The research in my group has been supported continuously since 1962 by the National Science Foundation Low Temperature Physics Program. NSF support has made it possible to undertake the sustained effort required to work effectively in low temperature physics.

The manuscript, in process for several years, was typed by Marirose Radostitz and Rachel Sewell. Most of the drawings were prepared by Sally Donovan. I have been fortunate in having such expert assistance.