

## CONTENTS

---

<i>Preface</i>	<i>page</i> vii
<b>1 Incidence Matrices</b>	1
1.1 Fundamental Concepts	1
1.2 A Minimax Theorem	6
1.3 Set Intersections	11
1.4 Applications	17
<b>2 Matrices and Graphs</b>	23
2.1 Basic Concepts	23
2.2 The Adjacency Matrix of a Graph	24
2.3 The Incidence Matrix of a Graph	29
2.4 Line Graphs	35
2.5 The Laplacian Matrix of a Graph	38
2.6 Matchings	44
<b>3 Matrices and Digraphs</b>	53
3.1 Basic Concepts	53
3.2 Irreducible Matrices	55
3.3 Nearly Reducible Matrices	61
3.4 Index of Imprimitivity and Matrix Powers	68
3.5 Exponents of Primitive Matrices	78
3.6 Eigenvalues of Digraphs	88
3.7 Computational Considerations	96
<b>4 Matrices and Bipartite Graphs</b>	107
4.1 Basic Facts	107
4.2 Fully Indecomposable Matrices	110
4.3 Nearly Decomposable Matrices	118
4.4 Decomposition Theorems	125
4.5 Diagonal Structure of a Matrix	136
<b>5 Some Special Graphs</b>	145
5.1 Regular Graphs	145

vi	Contents	
5.2	Strongly Regular Graphs	148
5.3	Polynomial Digraphs	157
<b>6</b>	<b>Existence Theorems</b>	164
6.1	Network Flows	164
6.2	Existence Theorems for Matrices	172
6.3	Existence Theorems for Symmetric Matrices	179
6.4	More Decomposition Theorems	184
6.5	A Combinatorial Duality Theorem	188
<b>7</b>	<b>The Permanent</b>	198
7.1	Basic Properties	198
7.2	Permutations with Restricted Positions	201
7.3	Matrix Factorization of the Permanent and the Determinant	209
7.4	Inequalities	214
7.5	Evaluation of Permanents	235
<b>8</b>	<b>Latin Squares</b>	250
8.1	Latin Rectangles	250
8.2	Partial Transversals	254
8.3	Partial Latin Squares	259
8.4	Orthogonal Latin Squares	269
8.5	Enumeration and Self-Orthogonality	284
<b>9</b>	<b>Combinatorial Matrix Algebra</b>	291
9.1	The Determinant	291
9.2	The Formal Incidence Matrix	293
9.3	The Formal Intersection Matrix	304
9.4	MacMahon's Master Theorem	310
9.5	The Formal Adjacency Matrix	317
9.6	The Formal Laplacian Matrix	324
9.7	Polynomial Identities	327
9.8	Generic Nilpotent Matrices	335
	<i>Master Reference List</i>	345
	<i>Index</i>	363