

Index

General topics concerning curves are listed under “curves.” Topics relating to special curves are listed under the name of the special curve. For example, things related to conics are listed under “conic,” topics relating specifically to ellipses are under “ellipse,” those concerning splines under “spline.” Much of the text discusses interpolation but only general items are listed under this heading. Most interpolation is listed under more specific headings for example: “basis construction,” “construction of curves” or “conic, through four points and touching a line.”

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