Victor Bryant

# *METRIC SPACES*

## Iteration and application

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# PREFACE

Some years ago I regularly gave a traditional course on metric spaces to second-year special honours mathematics students. I was then asked to give a watered-down version of the same material to a class of combined honours students (who were doing several subjects, including mathematics, at a more general level) but, to put it mildly, the course was not a success. It was impossible to motivate students to generalise real analysis when they had never understood it in the first place and certainly could not remember much of it. It was also counter-productive to start the course by revising real analysis because that convinced the students that this was 'just another analysis course' and their interest was lost for evermore.

So when I gave the course again the following year I decided to turn the material inside out and to *start* with the applications (namely the use of contractions in solving a wide range of equations). This meant that the first chapter was a revision of some iterative techniques used to obtain approximations to solutions of equations. This immediately captured the interest of the class: they enjoyed using their calculators and writing programs to solve the equations. Some of the ideas were entirely new to them; for example using iteration to solve an equation with constraints, or solving a differential equation by iterating with an integral and obtaining a sequence of functions.

The second and third chapters were more traditional but the big difference was that the need for distance, function space, closed set, and so on, had been anticipated and motivated. Another difference was that, having approached the subject via iteration, it was then natural to define all the concepts in terms of sequences: hence closed sets (rather than open ones) formed the basis of the approach.

For most students the fourth chapter was the highlight of the course. It consisted of the contraction mapping principle and the use of its algorithmic proof in solving equations. But I was then able to say to them that, as we had developed all the tools of the subject, it was now an easy matter to look back to real analysis and get a better understanding of it. The last chapter therefore recalled and generalised the classic theorems of real analysis.

This book, then, is an approach to metric spaces along those lines. It tries to avoid assuming that the reader knows much about analysis and if a difficult concept is to be encountered the reader is prepared by several glimpses of it, in examples, well in advance. My aim is to provide a book which can be read and enjoyed by a wide range of second- or third-year students in universities or polytechnics. The only prerequisite is to have done a course on elementary analysis: it is not a prerequisite to have understood it nor to have remembered it all.

There are several people who have contributed indirectly to this work. Firstly I thank Hazel Perfect and John Pym for their most constructive comments. Secondly my well-worn copies of the books on metric spaces by Copson, Simmons and Sutherland testify to the use which they have been to me over the years. Next I must mention my colleagues Mary Hawkins and Harry Burkill: at one time we collaborated on a metric space course, and for any of their examples and proofs which may remain in this new approach I thank them. Also I thank Anne Hall for preparing a beautiful typescript from my almost-illegible original. Finally, and principally, I thank my friend and mentor, Roger Webster. It was his M.Sc. course on functional analysis many years ago which rekindled my pleasure in mathematics and introduced me to metric spaces.

I have tried to provide a readable and natural introduction to an abstract subject in a down-to-earth manner. Even if I can transmit only a fraction of the pleasure which the subject has given me, then I will have been successful.

Sheffield
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Victor Bryant