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J. A. Hudson

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PREFACE

Although there has been a widespread change of emphasis in solid mechanics, in which the linearised theory of elasticity has partly given way to a more fundamental approach based on a generalised theory of continuum mechanics, there are still areas of physics (for instance seismology, noise analysis and the non-destructive testing of materials) in which the linearised theory of elastodynamics is of fundamental importance.

Very often, the most appropriate mode of analysis of a disturbance is in terms of normal modes. The Earth, for instance, is finite and since about 1960 the theory of the normal modes of oscillation of the Earth has taken over a substantial part of the subject of seismology. However, most seismic sources (natural or artificial) are short in duration, and the seismogram is most easily read in terms of compact pulses with well-defined arrival times. Normal mode analysis is clearly not appropriate in this case.

This book is an attempt to collect together the fundamental results of linearised elastodynamics primarily from the point of view of the propagation of transient pulses in an isotropic material which may be regarded as unbounded. This means, for instance, that in chapters 5 and 6, which contain an investigation into uniqueness, reciprocity, and representations in terms of Green functions, it is necessary to introduce 'conditions at infinity' or radiation conditions.

Analysis of surface waves and of the reflection and refraction of plane waves (in chapter 3) proceeds in terms of harmonic time-dependence, and chapter 6 consists entirely of an investigation of time-harmonic problems. However, there is no normal-mode theory here, except in so far as Love waves may be regarded as a sequence of normal modes.

Most of the results described here would usually be described as 'classical' and assigned a date somewhere between Poisson and

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PREFACE

Love. However some of them are surprisingly recent. For instance, although no doubt the idea of the ray path transmission of a disturbance in a solid was known in the last century, the theory (described in chapter 4) was first published in 1956. Representation in terms of Green functions originated with Stokes and was developed by Love using methods which depend essentially on the material being homogeneous. The generalised form of the result for inhomogeneous (and anisotropic) material does not appear to have been published until 1964.

The book is intended to be 'fundamental' in that it provides the ground work to enable anyone to tackle, say, diffraction and scattering problems but (with one exception) it does not attempt any such problems itself. The problem of a line source in a homogeneous half-space (the Lamb problem) is covered in chapters 7 and 8, but this is to demonstrate the generation of surface and interface waves, and to bring out the usual characterisation of the surface wave as the contribution from the residue at a pole in the complex wave number plane.

Chapter 1, on the derivation of the equations of motion and of continuity, is brief, since much of this material can be found in many other books. Thus some familiarity with the concepts of stress and strain has been assumed in the reader. In other respects, however, the policy has been to be as complete as possible with regard to formulae which might be useful for reference. So two-dimensional as well as three-dimensional sources are described, together with all components of each Green function. References have been given to enable the reader to pursue a particular subject in more detail than was possible to include here.

Finally, a chapter (9) on the theory of linear visco-elasticity has been included, since all 'elastic' waves are, in practice, somewhat damped. In addition, it was thought that the fundamentals of the subject could be dealt with while keeping the size of the book within reasonable bounds.

Many people have contributed to the writing of this book. Special thanks are due to Ralph Lapwood from whom I learned the subject in the first place and who read and made helpful comments on the early chapters. I am also very grateful to John Heritage for a very careful job of checking through the whole book.