

## CONTENTS

## Chapter I. Symmetries and Groups

1. Definitions	1
2. Examples	1
3. Groups	5
4. Symmetries of regular polygons	8
5. Presentations	12
6. Change of presentation	15

## Chapter II. Isometries of the Euclidean Plane

1. Geometric types of isometries	23
2. Structure of $E$	28
3. Representation of $E$	30
4. Stabilizers and transitivity	31
5. Similarity	33
6. The affine group	34

## Chapter III. Subgroups of the Group of Isometries of the Plane

1. Subgroups with discrete translation subgroup	39
2. Frieze groups	40
3. Discontinuous groups	43
4. Regular tessellations of the Euclidean plane	46

5. Triangle groups	51
6. Regular solids in three dimensional Euclidean space	52
7. Hyperbolic triangle groups	56
Chapter IV. Discontinuous Groups of Isometries of the Euclidean Plane: Plane Crystallographic Groups	
1. Introduction	61
2. The group $G^+$	63
3. Conjugation of $T$ by $\rho$	65
4. Enumeration of cases	67
5. Summary	74
Chapter V. Regular Tessellations in Higher Dimensions	
1. Introduction	78
2. The standard regular solids	79
3. Regular tessellations: examples	82
4. Regular tessellations: definitions	84
5. Existence	86
6. Duality	89
7. Regular tessellations of dimensions 2 and 3	90
8. Regular tessellations in dimensions $n \geq 4$	92
9. The 4-dimensional solid with 24 cells	94
10. The 4-dimensional solid with 600 cells	97
Chapter VI. Incidence Geometry of the Affine Plane	
1. Combinatorial description of the affine group	107
2. The coordinate plane over a field	111
3. Incidence planes	112

4. Introduction of coordinates	115
5. The automorphism group of an incidence plane	117
Chapter VII. Projective Geometry	
1. Introduction	121
2. Definition of the real projective plane	123
3. The projective plane over an arbitrary field	125
4. Coordinatization of projective planes	127
5. The projective group	130
Chapter VIII. Inversive Geometry	
1. Inversion in a circle	136
2. Geometric treatment of inversion	138
3. The inversive group	142
4. Classification of the elements of $M^+$	146
5. Solvable subgroups of $M^+$	149
6. Finite subgroups of $M^+$	152
7. Simplicity of $M^+$	155
Chapter IX. Hyperbolic Geometry	
1. The hyperbolic group and hyperbolic plane	160
2. Solvable subgroups of $H$	163
3. Incidence and angle	165
4. Combinatorial definition of area	169
5. The hyperbolic metric	171
6. Two more theorems	175

Chapter X. Fuchsian Groups	
1. Fundamental regions	179
2. Geometry of the fundamental region	183
3. Digression: elements of finite order	186
4. The Cayley tessellation	187
5. The Poincaré presentation	189
6. Combinatorial description of the presentation	192
7. Groups containing inversions	194
8. Poincaré's polygon theorem	194
9. Examples	195
10. Surface groups	200
11. Classification of Fuchsian groups	205
12. The Riemann-Hurwitz formula	206
References	214
Index	216