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Groups and geometry

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PREFACE

This book is intended as an introduction, demanding a minimum of background, to some of the central ideas in the theory of groups and in geometry. It grew out of a course, for advanced undergraduates and beginning graduate students, given several times at the University of Michigan and, in 1980-81, at the Université de Picardie. It is assumed that the reader has some acquaintance with the algebra of the complex plane, with analytic geometry, and with the basic concepts of linear algebra. No technical knowledge of geometry is assumed, and no knowledge of group theory, although some exposure to the fundamental ideas of group theory would probably prove helpful.

We exploit the well known close connections between group theory and geometry to develop the two subjects in parallel. Group theory is used to clarify and unify the geometry, while the geometry provides concrete and intuitive examples of groups. This has some influence on our emphasis, which is primarily combinatorial. The groups are mainly infinite groups, often given by generators and relations. The geometry is mainly incidence geometry; apart from linear algebra, we have used analytic and metric methods sparingly. In the interest of intuition we have, with one exception, confined attention to two-dimensional geometry.

Except in connection with projective geometry, we have paid little attention to axioms. We feel that this is no real loss of rigor, since, where intuition is not found sufficient, the reader can always fall back on analytic geometry to verify elementary assertions.

The emphasis has been on geometric spaces and their groups, rather than on theorems concerning special configurations. There are three chapters that lie outside our systematic progress through the study of the classical geometric spaces, and which may be regarded as optional. Each shows the general abstract ideas put to work on special problems. Chapter IV classifies the planar crystallographic groups. Chapter V classifies the regular tessellations of the higher dimensional spheres (and thus the regular solids) and Euclidean spaces, using very little beyond two-dimensional arguments. Chapter X, after developing basic concepts in the theory of Fuchsian groups, is mainly devoted to examples.

Throughout, while emphasizing abstract concepts, we have illustrated these concepts with concrete and intuitive examples, both in the text and in the problems that follow each chapter.

Where appropriate we have tried to guide the reader by references, both to collateral or alternative expositions and to sources for material beyond that presented here. A substantial list of references appears at the end of the book.

The author is grateful to his former students for their interest, criticism, and suggestions. He wants to thank his colleagues A. Boidin and A. Fromageot for invaluable assistance both with his course at Amiens and in the preparation of the resulting notes. He especially thanks Antoine Fromageot, without whose unfailing enthusiasm and imaginative hard work the precursor to the present book would not have been undertaken. He thanks David Tranah of the Cambridge University Press who has made the publication of this book as easy and pleasant as possible, and Wagner Associates, of Skokie, Illinois, for their enthusiastic and perfect work in preparing the manuscript.

Roger Lyndon

Ann Arbor, 31 August 1984

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