

INDEX

- action-angle variables, 48
- additional resonant terms
 - area-preserving planar map, 308
 - rational rotation, 86, 258
 - time-dependent vector field, 91
- Anosov automorphism
 - and Anosov diffeomorphisms, 134
 - chaotic basic set, 159
 - definition, 133
 - and dynamical systems, 120
 - homoclinic points, 136–8, 182
 - periodic points, 134
- Anosov diffeomorphism
 - conjugacy to automorphism, 134
 - and dynamical systems, 133
 - structural stability, 135
- area-preserving planar map
 - Birkhoff normal form, 305
 - complex form, 306
 - and Hénon area-preserving map, 169
 - normal form, 308
- Arnold's circle map, 248
 - and dissipative standard map, 349
 - and unfoldings of rotations, 257
- Arnold tongues
 - definition, 252
 - symmetry of, 293
 - and unfoldings of rotations, 257
- atlas, 2
- attracting set, 346
 - chaotic, 161
- attractor, 346
 - strange, 161
- Aubry–Mather Theorem, 338
- axiom- A diffeomorphism, 158
- Baker's transformation, 184
- basic sets
 - Anosov automorphism, 159
 - Decomposition Theorem, 158–9
 - horseshoe diffeomorphism, 158
 - spinning diffeomorphism, 160
- bifurcation, local, 191
- bifurcation curve, 207
- bifurcation point, 190
- biharmonic oscillator, 48
- Birkhoff attractor, 348
- Birkhoff normal form
 - for area-preserving planar map, 305
 - for Hamiltonian, 303
- Birkhoff periodic orbit, 336
- Birkhoff periodic points, 309
 - of type (p, q) , 336
- Birkhoff set, 347
 - rotation interval, 347
- Birkhoff Theorem, 338
- blowing-up
 - for cusp singularity, 107
 - directional, 105
 - polar, 102
 - in x -direction, 106
 - in y -direction, 106
- Bogdanov map, 359
- Bogdanov points, 378
- Calabi invariant, 345
- C^1 -norm for vector field, 123, 125
- canonical polar coordinate system, 305
- canonical transformation, 45
- Cantor set
 - definition, 333, 386
 - in double invariant circle bifurcation, 375
 - invariant
 - for area-preserving twist homeomorphism, 338
 - for circle homeomorphism, 332–5
 - for horseshoe diffeomorphism, 139–47; dynamics on, 149–54, 184
 - in Smale–Birkhoff Theorem, 165
 - for spinning diffeomorphism, 160
- centre eigenspace, 94
- centre manifold
 - differentiability of, 97

- centre manifold (*cont.*)
 - global, 96
 - local, 94
- Centre Manifold Theorem, 94
 - for family of vector fields, 200
- chaotic basic set
 - Anosov automorphism, 159
 - horseshoe diffeomorphism, 154, 158–9, 164
 - spinning diffeomorphism, 160
- chaotic behaviour, 244
- chart, 2
- circle map
 - diffeomorphism, 9
 - homeomorphism, 7
 - rotation, 6
- closed orbit
 - definition, 13
 - hyperbolic, 70
 - local structural stability, 124
 - stable/unstable manifolds, 71
- codimension, 79
 - 2×2 real matrices
 - eigenvalues zero/non-zero, 112
 - eigenvalues modulus unity, 247, 291, 292
 - saddle-node and Hopf singularities, 199
- complex form
 - of area-preserving map, 306
 - of planar diffeomorphism, 86
 - of planar differential equation, 91, 115
- complexity
 - Anosov diffeomorphism, 135, 136
 - Bogdanov map, 360–3
 - at generic elliptic point, 309, 339, 340
 - Hénon area-preserving map, 55, 309
 - and homoclinic tangles, 68, 136, 139
 - horseshoe diffeomorphism, 153, 164
 - logistic map, 231
 - repetition on all scales, 138, 164, 340
 - in symbolic dynamics, 153, 166
 - in weakly dissipative systems, 345–55
- conjugacy
 - C^k , 23
 - semi-, 333
 - topological (C^0), 20
- continued fraction, 342
 - principal convergent of, 342
- contraction
 - linear diffeomorphisms, 64
 - linear flows, 65
- cusp bifurcation, 206
 - algebraic criteria, 210
- cusp singularity, 108, 206
 - versal unfolding, 206
- δ -shadow orbit, 352
- degeneracy conditions
 - cusp singularity, 108
 - degenerate (generalised) Hopf singularity, 102, 221
 - non-degenerate Hopf singularity, 102
 - non-hyperbolic singularity on \mathbb{R}^2 , 79
- degeneracy conditions (*cont.*)
 - saddle-node singularity, 101
- degrees of freedom, 42
- Denjoy counterexample, 332–5
- Denjoy Theorem, 28
- derivative map
 - as representative of tangent map, 154–5
 - and transformation of vector fields, 24
- diffeomorphism
 - C^k , 2
 - on \mathbb{R}^n
 - linear hyperbolic, 64, 67; orientation preserving/reversing, 67
 - non-linear, 68–9; orientation preserving, 167
 - on S^1 , 6, 9, 26–8
- differentiable manifold, 1, 2
 - C^r , 4
 - dimension, 2
- $\text{Diff}^1(S^1)$, 132
- $\text{Diff}^1(T^n)$, 133, 135
- $\text{Diff}^1(U)$, 124
- double invariant circle bifurcation, 368–78
- double limit cycle bifurcation, 212–15, 274, 275
 - significance in vector field approximations, 290
- Duffing attractor, 161, 179, 180
- Duffing equation, 161, 176
 - Melnikov function for, 176–7
- ε - C^1 -close, 123
- ε - C^1 -perturbation, 123
- ε -neighbourhood
 - in $L(\mathbb{R}^n)$, 120
 - in $\text{Vec}^1(U)$, 123
- ε -pseudo-orbit, 352
- elliptic fixed point, 51, 305
 - generic, 308, 338–40
- equivalence
 - of local families
 - of diffeomorphisms, 193
 - of vector fields, 191
 - of two flows, 28–33
 - C^k , 30
 - topological (C^0), 29
- expansion
 - linear diffeomorphisms, 64
 - linear flows, 65
- exponential matrix, 65, 398
- family
 - local, 191
 - of vector fields/diffeomorphisms, 190
- first integral, 43
- fixed point
 - of diffeomorphism, 5
 - asymptotically stable, 5
 - marginally (neutrally) stable, 6
 - stable, 5
 - unstable, 6
 - of flow, 13
 - and singular points, 15
 - stability, 13

- flip bifurcation, 221–6, 244
 - in logistic map, 226–8
 - see also* period doubling bifurcation
- Floquet Theorem, 89, 114
- flow
 - definition, 12
 - and differential equations, 14
 - linear hyperbolic, 65
 - local, 15
- Flow-box Theorem, 26
- flow velocity, 1, 14
 - see also* vector field
- fold bifurcation, 218–19, 244
 - in logistic map, 230–1, 244
- fundamental matrix, 40
- generalised coordinates, 43
- generalised momenta, 43
- generating function for symplectic transformations, 304
- generic property, 119–21
- global (cross) section of flow
 - on manifold, 34
 - on torus, 35
- Hamilton's equations, 43
- Hamiltonian flow, 43
 - volume preserving property, 47
- Hamiltonian (function), 43
 - Birkhoff normal form, 303
- Hamiltonian system
 - conservative, 42
 - integrable, 47–8
 - non-integrable, 48
 - and area-preserving Poincaré maps, 49–54
- Hartman–Grobman Theorem
 - diffeomorphisms, 68
 - flows, 69
- Hénon area-preserving map
 - homoclinic points, 169
 - numerical experiment, 54–5
 - and vector field approximation, 317–19
- Hénon attractor, 162
- heteroclinic point to a periodic orbit, 357
- homeomorphism
 - charting, 2
 - conjugating, 20
 - definition, 2
- homoclinic point
 - to a fixed point
 - tangential, 180, 289, 361
 - transverse, 136
 - from Melnikov function, 170–6
 - to a periodic orbit
 - transverse, 165, 357
- homoclinic tangle
 - and embedded horseshoe maps, 165, 170
 - at a fixed point, 165–9
 - and non-trivial Birkhoff sets, 350–5, 358
 - at a periodic orbit, 165, 169–71, 339
 - and vector field approximations, 289–90
- homological equation
 - diffeomorphisms, 84
 - vector fields, 73
 - time periodic, 90
- Hopf bifurcation
 - for diffeomorphisms, 247, 253–62
 - generalised, 369
 - for vector fields, 203–6
 - generalised, 211–15
 - remote, 281–3
 - symmetric, 274
- Hopf Bifurcation Theorem
 - diffeomorphisms, 261
 - vector fields, 205
- Hopf singularity, 102
 - generalised of type l , 211
 - versal unfolding, 212
 - versal unfolding, 203
- horseshoe diffeomorphism
 - basic sets, 158–9
 - canonical example, 139–47
 - and dynamical systems, 120
 - homoclinic points, 164, 187
 - invariant Cantor set, 142–3, 149, 160, 183
 - complexity, 153–4, 183–4
 - conjugacy of map to left-shift, 153
 - dynamics of left-shift, 148–9
 - representation by symbol sequences, 149–53
- hyperbolic fixed point
 - diffeomorphism on manifold, 155–6, 402
 - diffeomorphism on \mathbb{R}^n
 - linear, 64
 - non-linear, 68
 - flow on \mathbb{R}^n
 - linear, 65
 - non-linear, 69; topological classification, 70
- hyperbolic (invariant) set, 157
- hyperbolic structure, 154
- induced local family, 192
- inevitable resonance
 - area-preserving planar map, 308
 - diffeomorphisms, 86, 255, 258
 - time-periodic vector fields, 92
- inset, 147
- invariant circle
 - of area-preserving map, *see* KAM (invariant) circle
 - of diffeomorphism, 71
 - Hopf, 259, 261, 360, 368–78
 - normally hyperbolic, 71
 - of flow, 282, 283, 286
 - see also* closed orbit, limit cycle
 - rotational, 345
- Invariant Manifold Theorem
 - diffeomorphisms, 68
 - flows, 70
- invariant set, 16
 - hyperbolic, 157
 - minimal, 16
 - negatively, 16

- invariant set (*cont.*)
 - positively, 16
- island chain, 52, 310, 339
 - dissipative, 371
- Jordan form
 - complex, 85
 - real, 93
- KAM (invariant) circle
 - at generic elliptic point, 339
 - golden (for standard map), 344
 - location using continued fractions, 342–3
- KAM iteration scheme, 326–30
- KAM Theorem, 319
- Katok Theorem, 338
- Kupka–Smale diffeomorphism, 164
- left-shift
 - and horseshoe diffeomorphism, 153
 - on m symbols, 183, 186
 - on two symbols
 - dense orbit, 148
 - dense periodic points, 148
 - periodic and aperiodic orbits, 147
- Leslie model attractor, 162
- level of degeneracy
 - area-preserving flow, 303
 - non-hyperbolic singularities on \mathbb{R}^2 , 79
 - cusp, 210
 - generalised Hopf, 211
 - Hopf, 199
 - saddle-node, 199
- Liapunov stability
 - diffeomorphisms, 5–6
 - flows, 13
- Lie bracket, 73
- lift
 - Anosov automorphism, 133
 - circle map, 7
 - twist homeomorphism, 335
- limit cycle, 20
- limit point, 17
- limit set, 17
- linear diffeomorphism
 - hyperbolic, 64
 - topological types, 67
- linear flow
 - algebraic types, 60
 - hyperbolic, 65–7
 - topological types, 33
- linear shear map, 269, 286
- linear vector field
 - hyperbolic, 65
 - topological type, 32–3
- linearisation
 - diffeomorphism, 68
 - vector field, 69
- Liouville Theorem, 45
- local coordinates
 - at bifurcation point, 191
- local coordinates (*cont.*)
 - at fixed point, 69
 - and manifolds, 2
 - at ordinary point, 24, 25
- local family, 191
 - of diffeomorphisms, 193, 215
 - of vector fields
 - equivalent, 191
 - induced, 192
 - versal, 196
- local flow, 15
- local (cross) section, 24
- logistic map, 226
 - flip bifurcation, 228
 - fold bifurcation, 231
 - period doubling cascade, 226–30
 - symbolic dynamics, 233
- Lorenz attractor, 163
- Malgrange Preparation Theorem, 196
- Mather Division Theorem, 236
- Mather set
 - definition, 336
 - for dissipative twist map, 349
 - rotation number of, 336
 - for standard map, 344
 - types of minimal, 338
- Mather Theorem for Anosov diffeomorphisms, 135
- Melnikov function, 170–6
 - definition, 174
 - for Duffing equation, 177
- monomial, 74
 - resonant, 81
- Morse–Smale systems, 133
- Moser Twist Theorem
 - analytic area-preserving map, 329
 - with finite differentiability, 330
 - and KAM Theorem, 319
- multidimensional saddle point, 67
- n - F system, 43
- noble numbers, 344
- non-degeneracy conditions
 - non-hyperbolic singularity on \mathbb{R}^2 , 79
 - cusp, 108
 - degenerate (generalised) Hopf, 102, 211
 - non-degenerate Hopf, 102
 - saddle-node, 101
- non-hyperbolic fixed point
 - and bifurcations, 190
 - diffeomorphisms on \mathbb{R}^2 , 245–7
 - flows on \mathbb{R}^2 , *see* singular point, non-hyperbolic
- non-wandering point, 16
- non-wandering set
 - Anosov automorphism, 159–60
 - definition, 17
 - horseshoe diffeomorphism, 158
 - hyperbolic and basic sets, 158
- normal form
 - diffeomorphism, 84–5

- normal form (*cont.*)
 - family of, 222, 253, 268, 294, 311
 - formal, 307
 - integrable Hamiltonian system, 48
 - vector field, 77
 - time-periodic, 91
- Normal Form Theorem
 - vector fields, 76
 - diffeomorphisms, 84
 - time-dependent, 91
- nowhere dense set, 333
- orbit
 - diffeomorphism, 5
 - flow, 12
- ordinary point of flow, 13
- outset, 147
- overlap map, 2
 - for differentiable manifold, 4
 - for symplectic manifold, 45
- parametric resonance, 42
- Peixoto Theorem
 - for circle diffeomorphisms, 132
 - for vector fields
 - closed disc, 126
 - compact 2-manifold, 127
- perfect set, 333
- period
 - of a closed orbit, 13
 - of a periodic orbit (cycle), 5
- period advance map, 38
 - and Poincaré map, 38, 40
 - and time-dependent normal forms, 89
- period doubling bifurcation, 226
 - logistic map, 230
- periodic non-autonomous system
 - elementary properties, 60–2
 - normal form, 89–91
 - and Poincaré maps, 38
- periodic orbit, 5
 - hyperbolic
 - on \mathbb{R}^n , 68–9
 - on manifold, 156–7
 - on S^1 , 10, 27
 - see also* Birkhoff periodic orbit
- periodic point
 - of diffeomorphism, 5
 - hyperbolic, 68
 - of flow, 13
- phase portrait, 13
- phase space, 1
- pitchfork bifurcation, 242, 244
- Poincaré–Bendixson Theorem, 19
- Poincaré–Birkhoff Theorem, 309
- Poincaré–Cartan invariant, 49
- Poincaré map, 34
 - area-preserving, 49
 - and Melnikov function, 172, 174–5
 - and period advance map, 38, 40
 - and suspension of diffeomorphism, 35
- Poisson bracket, 73
- product flow, 59, 395
- punctured plane, 103
- q -cycle, 5
- \mathbb{R} -action, 12
- regular point, 13
- residual set, 119, 164
- resonance
 - parametric, 42
 - strong/weak, 248, 260, 287
- resonance condition
 - diffeomorphisms, 84, 85, 258
 - Hamiltonians, 303–4
 - vector fields, 74, 77, 81
 - time-periodic, 91
- resonance tongue
 - Arnold, 252, 257, 293
 - and double invariant circle bifurcation, 373
 - and Hopf bifurcation in planar diffeomorphisms, 257–8
 - and unfolding of $(2\pi p/q)$ -rotation, $q \geq 5$, 259–61
 - from vector field approximation, 285–7
- resonant terms
 - diffeomorphisms
 - additional, 86
 - inevitable, 86
 - vector fields, 76
 - time-periodic: additional, 92; inevitable, 92
- Riemannian structure, 156
- right-shift, 183, 244
- Rössler attractor, 163
- rotation interval, 335
- rotation number
 - of circle homeomorphism, 26
 - on Hopf invariant circle, 255, 257–8, 260
 - internal/external, 346
 - irrational
 - and circle diffeomorphism, 28
 - and Mather set of Cantor type, 338
 - and perturbation of area-preserving twist map, 319
 - lifted, 308, 335
 - of a Mather set, 336, 387
 - of a periodic point, 336
 - of a point, 352
 - rational
 - and Arnold circle map, 248–53
 - and circle diffeomorphisms, 27
 - and perturbation of area-preserving twistmap, 309–10
 - and resonance tongue, 260
- saddle connection
 - and Melnikov function, 171, 176
 - and structural stability, 126–9
- saddle connection bifurcation
 - heteroclinic, 276–7, 279, 282
 - homoclinic, 208–9
 - remote, 282–3

- saddle connection bifurcation (*cont.*)
 - homoclinic (*cont.*)
 - symmetric, 274–5
 - significance in vector field approximation, 289–90
- saddle-node bifurcation
 - for diffeomorphisms, 245–6
 - for vector fields, 199–200
 - algebraic criteria for, 202–3
 - symmetric, 272, 274
 - and tangency of isoclines, 200–2
- saddle-node singularity, 101
 - versal unfolding, 200
- Sarkovskii Theorem, 230
- Seifert foliation, 264
 - averaging in, 265, 295
- semi-conjugacy, 333
- sensitive dependence on initial conditions, 168, 169, 244
- shadowing lemma, 352
- singular point
 - hyperbolic, 69
 - non-hyperbolic on \mathbb{R}^2 , 79
 - normal forms at, 80–3
 - non-simple, 69
 - of a vector field, 15
 - and fixed point of flow, 15
- singularity, *see* singular point *and* cusp-, Hopf- *and* saddle-node singularities
- Smale–Birkhoff Theorem, 165
- small denominator problem, 321
- spectral norm, 399
- spectral radius, 399
- spinning diffeomorphism, 160
- stable eigenspace, 65
- stable manifold
 - of a closed orbit, 71
 - of a fixed point
 - diffeomorphism, 68
 - flow, 70
- standard map, 340
 - dissipative, 349
- state space, 1
- state transition matrix
 - definition, 40
 - elementary properties of, 61
 - and period advance/Poincaré map, 40, 89
- stereographic projection, 3, 56, 126, 144
- strange attractor, 161
 - examples, 161–4
- strong resonance, 248, 260, 287
- structural stability
 - Anosov diffeomorphisms, 135
 - diffeomorphisms on S^1 , 132
 - flows on compact 2-manifold, 127
 - flows/diffeomorphisms in $L(\mathbb{R}^n)$, 121
 - and hyperbolicity, 121
 - local, 124
 - non-compact sets, 128–31
 - overview, 119–20
 - vector fields, 125, 126
- suspension
 - of diffeomorphism, 36
 - of family, 245
- symbolic dynamics
 - horseshoe diffeomorphism, 149–54, 183–4
 - logistic map, 231–4
 - on m symbols, 183, 186
 - on two symbols, 147–9, 184, 244
 - symbol sequence, 147–9, 183
- symplectic diffeomorphism, 45
- symplectic manifold, 45
- symplectic transformation, 45
- tangent map, 154
- tangent space, 14, 154
 - metric for, 155–6, 185
- topological (C^0) conjugacy, 20
- topological (C^0) equivalence, 29
- topological type
 - of a fixed point, 64
 - of a flow, 32
 - k -jet determination of, 102
 - of non-hyperbolic fixed point of a flow, 72
 - and blowing-up, 102–8
 - and centre manifold, 93, 99
- trajectory
 - of diffeomorphism, 5
 - of flow, 12
- transcritical bifurcation, 219–21, 244
- trapping hypothesis, 347
 - and dissipative standard map, 349
- trapping region, 346
- twist interval, 335
- twist map
 - area-preserving, 308
 - homeomorphism, 335; and Mather sets, 338
 - and KAM Theorem, 326
 - and vector field approximations, 317
 - diffeomorphism, 345
 - dissipative, 345; and Mather sets, 347–9
 - homeomorphism, 335
- unfolding
 - of diffeomorphism, 193
 - equivariant, 271
 - $(2\pi/q)$ -symmetric, $q=4$, 276
 - of vector field, 191
 - see also* versal unfolding
- unstable eigenspace, 65
- unstable manifold
 - of closed orbit, 71
 - of fixed point
 - diffeomorphism, 68
 - flow, 70
- $\text{Vec}^1(D^2)$, 125
- $\text{Vec}_{\text{in}}^1(D^2)$, 126
- $\text{Vec}^1(M)$, 126
- $\text{Vec}^1(U)$, 123

- vector field
 - approximations
 - to area-preserving planar map, 311
 - to planar diffeomorphisms, 262–71
 - complex form, 91, 115
 - and flow, 1, 14
 - Hamiltonian, 174
 - k -jet, 101
 - linear
 - algebraic types, 60
 - topological types, 32–3
 - singular point of, 15
 - velocity field, *see* vector field
- versal unfolding, 196
 - equivariant, 271, 287
 - $(2\pi/q)$ -symmetric; $q=2$, 272; $q=3$, 275; $q \geq 5$, 282
 - mini-, 196, 198, 199
- wandering point, 158, 166
- weak focus, 102
- weak node, 99
- weak resonance, 248, 260, 287
- Weak Resonance Theorem, 260
- \mathbb{Z} -action, 12