

Cambridge University Press
978-0-521-31650-7 - An Introduction to Dynamical Systems
D. K. Arrowsmith and C. M. Place
Frontmatter
[More information](#)

An introduction to dynamical systems

Cambridge University Press
978-0-521-31650-7 - An Introduction to Dynamical Systems
D. K. Arrowsmith and C. M. Place
Frontmatter
[More information](#)

To
VLADIMIR IGOREVICH ARNOLD
and
STEPHEN SMALE
for their inspirational work

Cambridge University Press
978-0-521-31650-7 - An Introduction to Dynamical Systems
D. K. Arrowsmith and C. M. Place
Frontmatter
[More information](#)

An introduction to

DYNAMICAL SYSTEMS

D. K. ARROWSMITH
*Lecturer, School of Mathematical Sciences,
Queen Mary & Westfield College, University of London*

C. M. PLACE
*Lecturer (formerly Department of Mathematics,
Westfield College, University of London)*



Cambridge University Press
978-0-521-31650-7 - An Introduction to Dynamical Systems
D. K. Arrowsmith and C. M. Place
Frontmatter
[More information](#)

Published by the Press Syndicate of the University of Cambridge
The Pitt Building, Trumpington Street, Cambridge CB2 1RP
40 West 20th Street, New York, NY 10011-4211, USA
10 Stamford Road, Oakleigh, Melbourne 3166, Australia

© Cambridge University Press, 1990

First published 1990
Reprinted 1991, 1994

British Library cataloguing in publication data

Arrowsmith, D.K.

An introduction to dynamical systems.

1. Differentiable dynamical systems

I. Title II. Place, C.M.

514'.7

Library of Congress cataloguing in publication data

Arrowsmith, D. K.

An introduction to dynamical systems / D.K. Arrowsmith and C.M. Place

p. cm.

Bibliography: p.

Includes index.

ISBN 0 521 30362 1. – ISBN 0 521 31650 2 (paperback)

1. Differentiable dynamical systems. I. Place, C. M. II. Title.

QA614.8.A77 1990

515'.352-dc20 89-7191 CIP

Transferred to digital printing 2001

CONTENTS

Preface		
1	Diffeomorphisms and flows	1
1.1	Introduction	1
1.2	Elementary dynamics of diffeomorphisms	5
1.2.1	Definitions	5
1.2.2	Diffeomorphisms of the circle	6
1.3	Flows and differential equations	11
1.4	Invariant sets	16
1.5	Conjugacy	20
1.6	Equivalence of flows	28
1.7	Poincaré maps and suspensions	33
1.8	Periodic non-autonomous systems	38
1.9	Hamiltonian flows and Poincaré maps	42
	Exercises	56
2	Local properties of flows and diffeomorphisms	64
2.1	Hyperbolic linear diffeomorphisms and flows	64
2.2	Hyperbolic non-linear fixed points	67
2.2.1	Diffeomorphisms	68
2.2.2	Flows	69
2.3	Normal forms for vector fields	72
2.4	Non-hyperbolic singular points of vector fields	79
2.5	Normal forms for diffeomorphisms	83
2.6	Time-dependent normal forms	89
2.7	Centre manifolds	93
2.8	Blowing-up techniques on \mathbb{R}^2	102
2.8.1	Polar blowing-up	102
2.8.2	Directional blowing-up	105
	Exercises	108
3	Structural stability, hyperbolicity and homoclinic points	119
3.1	Structural stability of linear systems	120
3.2	Local structural stability	123
3.3	Flows on two-dimensional manifolds	125
3.4	Anosov diffeomorphisms	132

Contents

3.5 Horseshoe diffeomorphisms	138
3.5.1 The canonical example	139
3.5.2 Dynamics on symbol sequences	147
3.5.3 Symbolic dynamics for the horseshoe diffeomorphism	149
3.6 Hyperbolic structure and basic sets	154
3.7 Homoclinic points	164
3.8 The Melnikov function	170
Exercises	180
4 Local bifurcations I: planar vector fields and diffeomorphisms on \mathbb{R}	190
4.1 Introduction	190
4.2 Saddle-node and Hopf bifurcations	199
4.2.1 Saddle-node bifurcation	199
4.2.2 Hopf bifurcation	203
4.3 Cusp and generalised Hopf bifurcations	206
4.3.1 Cusp bifurcation	206
4.3.2 Generalised Hopf bifurcations	211
4.4 Diffeomorphisms on \mathbb{R}	215
4.4.1 $D_x f(0) = +1$: the fold bifurcation	218
4.4.2 $D_x f(0) = -1$: the flip bifurcation	221
4.5 The logistic map	226
Exercises	234
5 Local bifurcations II: diffeomorphisms on \mathbb{R}^2	245
5.1 Introduction	245
5.2 Arnold's circle map	248
5.3 Irrational rotations	253
5.4 Rational rotations and weak resonance	258
5.5 Vector field approximations	262
5.5.1 Irrational β	262
5.5.2 Rational $\beta = p/q$, $q \geq 3$	264
5.5.3 Rational $\beta = p/q$, $q = 1, 2$	268
5.6 Equivariant versal unfoldings for vector field approximations	271
5.6.1 $q = 2$	272
5.6.2 $q = 3$	275
5.6.3 $q = 4$	276
5.6.4 $q \geq 5$	282
5.7 Unfoldings of rotations and shears	286
Exercises	291
6 Area-preserving maps and their perturbations	302
6.1 Introduction	302
6.2 Rational rotation numbers and Birkhoff periodic points	309
6.2.1 The Poincaré–Birkhoff Theorem	309
6.2.2 Vector field approximations and island chains	310
6.3 Irrational rotation numbers and the KAM Theorem	319
6.4 The Aubry–Mather Theorem	332
6.4.1 Invariant Cantor sets for homeomorphisms on S^1	332
6.4.2 Twist homeomorphisms and Mather sets	335
6.5 Generic elliptic points	338
6.6 Weakly dissipative systems and Birkhoff attractors	345

Cambridge University Press
978-0-521-31650-7 - An Introduction to Dynamical Systems
D. K. Arrowsmith and C. M. Place
Frontmatter
[More information](#)

Contents

6.7 Birkhoff periodic orbits and Hopf bifurcations	355
6.8 Double invariant circle bifurcations in planar maps	368
Exercises	379
Hints for exercises	394
<i>References</i>	413
<i>Index</i>	417

PREFACE

In recent years there has been a marked increase of research interest in dynamical systems and a number of excellent postgraduate texts have been published. This book is specifically aimed at the interface between undergraduate and postgraduate studies. It is intended both to stimulate the interest of final year undergraduates and to provide a solid foundation for postgraduates who intend to embark on research in the field. For example, a challenging third-year undergraduate course can be constructed by selecting topics from the first four chapters. Indeed, lecture courses taught by one of us (CMP) provided the basis for Chapters 1, 2 and 4. On the other hand, Chapter 6 is directed at first-year postgraduate students. It contains a selection of current research topics that illustrate the interaction between superficially different research problems.

A major feature of the book is its extensive set of exercises; more than 300 in all. These exercises not only illustrate the topics discussed in the text, but also guide the reader in the completion of technical details omitted from the main discussion. Detailed model solutions have been prepared and hints to their construction are provided.

The reader is assumed to have attended courses in analysis and linear algebra to second-year undergraduate standard. Prior knowledge of dynamical systems is not necessary; however, some familiarity with the qualitative theory of differential equations and Hamiltonian dynamics might be an advantage.

We would like to thank Martin Casdagli for sharpening our understanding of Birkhoff attractors, David Knowles and Chris Norman for helpful discussions and Carl Murray for steering some awkward diagrams to a laser printer. We are grateful to the *Quarterly Journal of Applied Mathematics* and Springer-Verlag for allowing us to use diagrams from some of their publications and our thanks go to Sandra Place for her fast and accurate typing of much of the manuscript. One of us (CMP) would like to acknowledge the Brayshay Foundation for its financial support throughout this project. Finally, we must both pay tribute to the patience and support of our families during the long, and often difficult, gestation period of the manuscript.