1

Mathematics

With this volume we enter on the second part of our study of science and civilisation in early China. Since mathematics and a mathematical approach to theories have been the backbone of modern science, it seems proper that this subject should come first in any description of Chinese science and technology. Western opinions of Chinese mathematics have often oscillated between two extremes – either extolling Chinese achievements to the point of exaggeration, or denigrating them by saying that mathematically the Chinese never did anything worth while, such knowledge as they had being transmitted from the Greeks. Yet, as will now become evident, the second opinion is certainly far from the truth.

THE WRITING OF NUMERALS, PLACE-VALUE AND ZERO

Table 19 shows the various forms which Chinese written numerals can take, or have taken in the past. Of these, the 'accountants' forms' came into use gradually during and after the Han (first century B.C.), being considered more elegant and less liable to falsification. In column A the lower digits are pictographs, but from the number 4 onwards there seems to have been a borrowing of similar sounding words (homophones) from botanical and zoological nomenclature. The oracle-bone inscriptions (fourteenth to eleventh centuries B.C.) and inscriptions on coins and bronze vessels (tenth to third centuries B.C.), given in columns C and D, are in some cases similar to those related to the 'counting-rod' characters, columns F and G, which have been supposed to originate from actual counting-rods laid out on a flat board. All later notations follow the counting-board system.

It has been said that the earliest book in which the rod-numerals appear is the *Wu Tshao Suan Ching* (Mathematical Manual of the Five Government Departments), written in the fifth, or perhaps the fourth century A.D., but no editions of the book seen by us show rod-numerals; the calculations are merely written out in the standard way. But the question

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Table 19. *Ancient and medieval Chinese numeral signs*

	A Standard modern forms		B Accountants' forms	C Shang oracle-bone forms (14th to 11th centuries B.C.)	D Bronze and coin forms (10th to 3rd centuries B.C.)	E Other forms found on coins of Chou period (6th to 3rd centuries B.C.)	F Counting-rod forms (2nd century B.C. to 4th century A.D.) units	F tens	G Late counting-rod forms (13th century A.D.) units	G tens	H Commercial forms (from 16th century A.D.)
1	一	*i*	弌 or 壹	一	一	一 丨	𝍩	𝍠	𝍠	𝍩	〡
2	二	*erh*	弍 or 貳	二	二	二 ‖	𝍪	𝍡	𝍡	𝍪	〢
3	三	*san*	叁	三	三	三 Ⅲ	𝍫	𝍢	𝍢	𝍫	〣
4	四	*ssu*	肆	亖	亖	亖 ㄨⅢ [illegible]	𝍬	𝍣	𝍣X	𝍬X	〤
5	五	*wu*	伍	㐅	㐅	𝍭 X X	𝍭	𝍤	𝍣〇	𝍬〇	〥
6	六	*liu*	陸	∧ 介	介	介 介 ⊥ ⊤	𝍮	𝍥	𝍥	𝍮	〦
7	七	*chhi*	柒	十	十	[illegible]	𝍯	𝍦	𝍦	𝍯	〧
8	八	*pa*	捌	)(	)(	X 𝍰	𝍰	𝍧	𝍧	𝍰	〨
9	九	*chiu*	玖	[illegible]	九	[illegible] 𝍱	𝍱	𝍨	𝍧X	𝍯X	〩 夂
10	十	*shih*	拾	丨	丨	十 ϕ					十
100	佰	*pai*	佰	See Table 20	See Table 20	[illegible]	Indicated by place		Indicated by place		[illegible]
1,000	仟	*chhien*	仟			千					千
10,000	萬	*wan*	萬			万					万
0	零	*ling*	零			Blank space until 8th century A.D.			〇		〇

turns out to be of little significance, for the printing of mathematical works began in the eleventh century A.D., and since there is ample evidence that rod-numerals had been in use more than a thousand years earlier, it must have depended on individual editors whether or not rod-numerals were used in any particular printed book. Moreover, Han mathematical texts certainly use expressions which imply the use of counting-rods.

On the other hand, an enigmatic representation in the *Tso Chuan* (Master Tsochhiu's Enlargement of the Spring and Summer Annals) under the date 542 B.C. has often been cited to show that rod-numerals go back to the middle of the Chou. The passage certainly shows an understanding of the place-value of digits,* but in view of the subsequent remodelling of the *Tso Chuan* text it would perhaps be unsafe to accept this as evidence for a period earlier than that of the Warring States for which, in any case, coins bear witness. If, as there is reason to think, the Chinese character for calculation, *suan* (筭), is a truly ancient pictograph of counting-rods, the numerals (as well as the counting-board) may go back to the first millennium B.C. Some of the oracle-bone numerals (column C), especially 5, 6, 7 and 10, certainly look like arranged counting-rods.

During the Chin and the Han, the functions of the two kinds of numerals such as 𝍦 and 𝍯 were stabilised. The former were used for units, the latter for tens, the former for hundreds, the latter for thousands, and so on. By the end of the third century A.D. at least, they were termed respectively *tsung* and *hêng* numerals. The *Sun Tzu Suan Ching* (Master Sun's Mathematical Manual) of this period says:

> In making calculations we must first know the positions (and structure) (*wei*) (of numerals). The units are vertical and the tens horizontal, the hundreds stand while the thousands lie down; thousands and tens therefore look the same, as also the ten thousands and the hundreds ... When we come to 6, we no longer pile up (strokes), and the five has not got a one (a ligature).

The system was thus stabilised as follows:

	1	2	3	4	5	6	7	8	9
Units Hundreds Ten thousands	𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨
Tens Thousands	𝍩	𝍪	𝍫	𝍬	𝍭	𝍮	𝍯	𝍰	𝍱

* The place-value of digits will be something familiar to every reader, if not by name at least from experience. Thus 1 is one, 10 (i.e. with the digit 1 moved one place to the left) indicates ten, 100 (the 1 two places to the left) one hundred, and so on. Decimals, of course, use place values to the right: thus 0·1 is one-tenth, 0·01 one-hundredth, etc.

Thus, for example, the number 4716 appeared as 𝍫 𝍦 𝍩 𝍥. The separation of neighbouring powers of ten in this way facilitated the use of counting-boards without marked vertical columns. But it is worth noting the striking fact that, as far back as the oracle-bone numerals of the thirteenth century B.C., the symbols for 1 and 10 were both straight lines, the former horizontal and the latter vertical. This was just the opposite of the convention recorded by Master Sun, but the principle was identical.

The word *wei* in the quotation from the *Sun Tzu* referred essentially to the positions of the rods in the columns on a counting-board, in other words to place-value. Another word was 'rank' (*têng*, 等). Before the eighth century A.D., the place where a zero would now be written was always left vacant. For example, in a Thang manuscript from the Tunhuang cave-temples, one roll contains multiplication tables; here the results appear as rod-numerals, and 405 is shown as 𝍫 𝍤.

The circular symbol for zero is first found in A.D. 1247 in print in the *Shu Shu Chiu Chang* (Mathematical Treatise in Nine Sections) of Chhin Chiu-Shao, but many have believed that it was in use during the preceding century at least. The usual view is that it was derived from India, where it first appears on the Bhojadeva inscriptions at Gwalior dated A.D. 870. But there is no positive evidence for this transmission, and the form could perhaps have been borrowed from the philosophical diagrams of which the twelfth-century A.D. Neo-Confucians were so fond. In any case, the Sung mathematicians had at their disposal a fully developed notation, as in this example from the work of Chhin Chiu-Shao, where the subtraction

$$1{,}405{,}536 = 1{,}470{,}000 - 64{,}464$$

appears as follows:

However, while the first written evidence for the zero in India is of the late ninth century A.D., it has been discovered in use about two hundred years earlier in Indo-China and other parts of south-east Asia. This fact may be of much significance. The literary and written evidence for place-value in India has been conflicting. A dating of the eighth century A.D. seems the earliest that can be determined with any certainty. But Indo-Chinese inscriptions use place-values much earlier – A.D. 609 in Champa in eastern India, and A.D. 605 in Cambodia. And it was soon after this, in 683, that the first inscriptions showing zero appear simultaneously in Cambodia and Sumatra. The Indian numerals, with separate signs for 10 and its multiples, were no improvement on the Greek and Hebrew alphabetical numbers, so Indo-China seems at first sight rather an unlikely place for such a revolutionary discovery. It is possible, though, that the written zero symbol, and the more reliable calculations which it permitted, really originated in the eastern zone of Hindu culture where it met the southern zone of the culture of the Chinese.

What stimulus could have been received at this interface? Could the culture there have adopted a zero sign from the empty blanks left for zero on the Chinese counting-boards? The essential point is that the Chinese had possessed, long before the time of the *Sun Tzu Suan Ching* (late third century A.D.), a place-value system which was fundamentally decimal. It may then be that the 'emptiness' of Taoist mysticism, no less than the 'void' of Indian philosophy, contributed to the invention of a symbol for zero. The first appearance of zero in dated inscriptions on the borderline of the Indian and Chinese culture areas would then seem hardly likely to be a coincidence.

As far as place-value is concerned, the Chinese appear always to have used it. The Shang numeral system (thirteenth century B.C.) is the earliest of which we have evidence; its symbols are given in Table 20, and the significance of place-value is obvious. Clearly, it was more advanced than contemporary scripts in Babylonia and Egypt. Admittedly, all three systems started a new cycle of signs at 10 and multiples of 10, and each had a place-value component, but only the Shang Chinese were able to express any number, however large, using no more than 9 numerals and a counting-board. This was a great stride forward. What is more, they never used the somewhat cumbersome Roman subtractive place-value system of forming numerals (where, for instance, 9 is written IX). The Shang Chinese system seems, then, the simplest of the ancient methods, and appeared two thousand years before the West inherited what are usually called the 'Arabic' numerals.

The other question we must glance at concerns the origin and history of the Chinese written form for the word zero, *ling* (零). The ancient meaning of this word was the last small raindrops of a storm, or drops of rain remaining afterwards on things. Later it came to be applied to any remainder, especially when coupled with another word, and would appear in a phrase like 'five over the hundred'. From this the transition of the use of *ling* for expressing the zero in a number such as 105 may readily be understood. Nevertheless, this use seems to have risen late, and though to determine precisely when would require a special investigation, we have never met with its use as a word meaning zero in a mathematical text until the Ming (fourteenth century A.D.). On the other hand, the algebraists in Sung times (thirteenth century A.D.) used the symbol o extensively, as we have seen (page 4), and it is easy to find examples of numbers written out in which the term *ling* could have been employed. It is, then, a little difficult to explain why the use of *ling* came in during the Ming and not at any other

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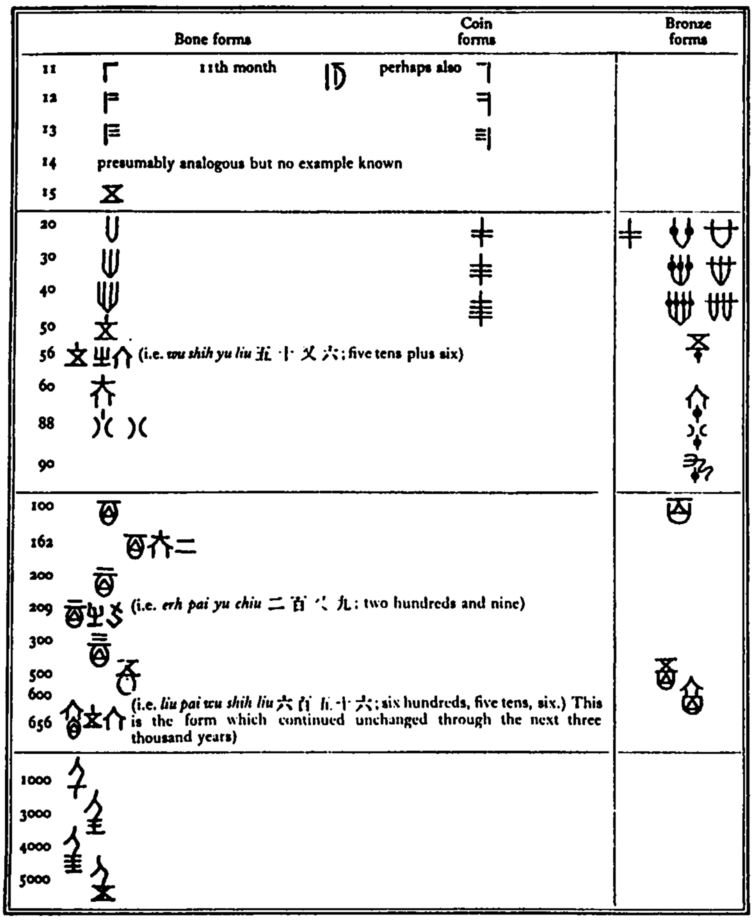
Table 20. *Notations for numerals higher than 10 on the Shang oracle-bones and on Chou bronze inscriptions*



time. However, we have suggested that perhaps the zero symbol had been pronounced 'ling' from the time of its first general use in the Sung, and it may well be that the use of the old character arose not only because it had long meant 'remainder', but also because the o symbol was shaped like a spherical raindrop.

SURVEY OF PRINCIPAL LANDMARKS IN CHINESE MATHEMATICAL LITERATURE

We give here a very brief summary of the most important books on mathematics produced by the Chinese over the centuries. Much Chinese mathematical literature was devoted to commentaries on them, and these often included original material.

From antiquity to the San Kuo period (third century A.D.)

Tradition has it that the *Chou Pei Suan Ching* (The Arithmetical Classic of the Gnomon and the Circular Paths of Heaven) is the earliest mathematical classic, but the first firm dates connected with it are some two centuries later than those associated with the *Chiu Chang Suan Shu* (Nine Chapters on the Mathematical Art). However, the question is a difficult one, and it will be convenient if we accept the traditional view here, especially since most of the book is so archaic that it is difficult not to believe that it goes back to the period of the Warring States (see chronological table, page 388).

The *Chou Pei* mentions Pythagoras' Theorem about right-angled triangles (the theorem that the square on the hypotenuse of the triangle is equal to the sum of the squares on the other two sides); this is at least as old as the sixth century B.C., but there is no proof of the kind later offered by Euclid (third century B.C.). It mentions the gnomon, a vertical stake that casts shadows the length and direction of which give information about the sun's altitude, and which was much used in early astronomy and calendar-making. There are also diagrams of stars close to the Pole Star and some other astronomical matters. Yet although concerned so much with astronomy, the book is interesting mathematically since there is some use of fractions, and discussion of their multiplication and division and the finding of common denominators. And even if the process of working out square roots is not given, the text makes it clear that square roots were certainly used.

The discussion of the right-angled triangle (Fig. 30), which occurs at the beginning of the book, is the oldest part of the text. A section of this runs as follows:

(1) Of old, Chou Kung addressed Shang Kao, saying, 'I have heard that the Grand Prefect (Shang Kao) is versed in the art of numbering. May I venture to enquire how Fu-Hsi anciently established the degrees of the celestial sphere? There are no steps by which one may ascend the heavens, and the earth is not measurable with a foot-rule. I should like to ask you what was the origin of these numbers?'

(2) Shang Kao replied, 'The art of numbering proceeds from the circle (*yuan*) and the square (*fang*). The circle is derived from the square and the square from the rectangle (lit. T-square or carpenter's square; *chü*).

(3) The rectangle originates from (the fact that) $9 \times 9 = 81$ (i.e. the multiplication table or the properties of numbers as such).

(4) Thus, let us cut a rectangle (diagonally), and make the width (*kou*) 3 (units) wide, and the length (*ku*) 4 (units) long. The diagonal (*ching*) between the (two) corners will then be 5 (units) long. Now after drawing a square on this diagonal, circumscribe it by half-rectangles like that which has been left outside, so as to form a (square) plate. Thus the (four) outer half-rectangles of width 3, length 4, and diagonal 5, together make (*tê chhêng*) two rectangles (of area 24); then (when this is subtracted from the square plate of area 49) the remainder (*chang*) is of area 25. This (process) is called 'piling up the rectangles' (*chi chü*[2]).

(5) The methods used by Yü the Great in governing the world were derived from these numbers.'

It will be remembered that the legendary Yü was the patron saint of hydraulic engineers and all those concerned with water-control, irrigation and conservancy. Epigraphic evidence from the Later Han, when the *Chou Pei* had taken its present form, shows us, in reliefs on the walls of the Wu Liang tomb-shrines (*c.* + 140), the legendary culture-heroes Fu-Hsi and Nü-Kua holding a carpenter's square and *quipu* (see Fig. 17 in volume 1 of this abridgement, page 62). The reference to Yü here undoubtedly indicates the ancient need for mensuration and applied mathematics.

Although no further commentary is needed, it is worth noting what seems to be one deeply significant point, namely the statement in paragraph

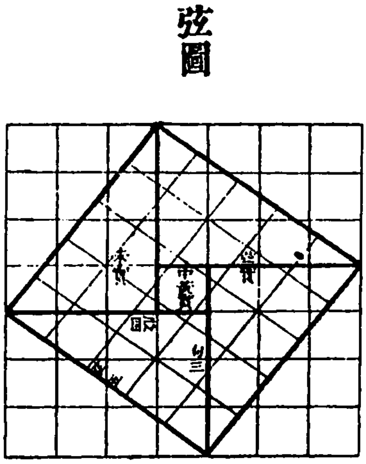


Fig. 30. The proof of Pythagoras' Theorem in the *Chou Pei Suan Ching*.

(3) that geometry arises from measurement. This seems to show the Chinese arithmetical-algebraic mind at work from the earliest times; they had little interest in abstract geometry, independent of concrete numbers, and consisting of theorems and propositions capable of proof, given only certain fundamental postulates to begin with. The precise numbers might be unknown, but numbers there had to be. And it is worth remembering that although the book was written at a time when astrology and divination were universally dominant, it speaks of the phenomena of the heavens and the earth without the slightest admixture of superstition.

The *Chiu Chang Suan Shu* represents a much more advanced state of mathematical knowledge than the *Chou Pei*. Dating the *Chiu Chang* is also difficult, and it is perhaps safest to regard it as a Chhin and Former Han book with Later Han accretions, bringing it down to some time in the second or early third century A.D. Perhaps the most influential of all Chinese mathematical books, it contains nine chapters with a total of 246 problems. These concern land surveying, engineering, the fair distribution of taxation, and other subjects, all of which bring various mathematical operations into play. The handling of fractions, the use of 'arithmetical' and 'geometrical' progressions of numbers, and the solution of simple simultaneous equations are all here, while the book also introduces the Rule of Three (a method of finding a fourth number from three given numbers where the ratio between two of them is the same as that between the third and the unknown fourth), and the Chinese invention, the Rule of False Position, for solving simple equations (page 49). Moreover, in the discussion on simultaneous equations, the problem of finding five unknowns with only four equations is discussed – a foreshadowing of indeterminate equations (page 49). In addition to the algebra contained in it, the *Chiu Chang* deals with geometrical questions – the areas of various figures and the volume of different solids, as well as the right-angled triangle and the famous problems associated with it, one of which is illustrated in Fig. 31. Some of these problems later found their way to Europe by way of India.

Apart from the two works described, there were many other mathematical books current during the Han dynasty; indeed the titles of some of them are known. Unfortunately all were afterwards lost. However, one Han book of considerable importance was the *Shu Shu Chi I* (Memoir on Some Traditions of Mathematical Art) by Hsü Yo, who lived and worked around A.D. 190. We know of it owing to a commentary written some four centuries later by Chen Luan, and it was clearly a very different kind of book from those already mentioned, being much nearer to Taoism and divination. Nevertheless it contains one of the earliest literary references to a magic square – a discovery in the theory of numbers which we shall consider on page 18 – and the earliest mention of the abacus.

折抵地爲弦以句及股弦并求股故先令句自乘見矩
冪令如高而一凡爲高一丈爲股弦并之以除此冪得
差所得以減竹高而半其餘即折者之高也此率與係
索之類更相返覆也亦可如上術令高自乘爲股弦并
冪去本自乘爲矩冪減之餘爲實倍高爲法則得折之
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去根如勾折處
如股折梢如弦
通長如股弦和

股弦和與勾求股法曰勾自乘爲實變股弦較乘股弦
和如股弦和而一正除得股弦較以減股弦和餘二段

Fig. 31. The problem of the Broken Bamboo (from Yang Hui's *Hsiang Chieh Chiu Chang Suan Fa*, A.D. 1261).