Applied production analysis
Applied production analysis
A dual approach

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For
MY PARENTS
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Preface

This book's subject matter is the economics of production. What differentiates this book most markedly from other books on production economics is that it relies almost solely on the use of "dual" techniques. Since the late 1960s, developments in duality theory and in its application to microeconomic data have radically transformed applied production analysis. Where before, most analyses focused on estimation of production or transformation functions, the use of dual cost or profit functions now dominates applied production economics. To some extent, this dominance is based on the "newness" of the dual approach, which makes many people anxious to learn about it and use it. But it also is quite apparent that the dual approach is often simply much easier to use and to interpret. And although dual treatments now dominate the journal literature and many excellent monographs on duality exist, as this book is being completed, a unified treatment of the basic matter of production economics from a dual perspective that is readily accessible to graduate students and nonspecialists does not exist. This book attempts to close this gap.

The book mimics a semester-long course on production economics that I have been teaching at the University of Maryland since 1979. Of course, in that time, many topics that I once thought important and spent a good deal of class time on gradually became either less interesting to me or so transparently useless that they were dropped from the course. What remains represents, I hope, a reasonable survey of producer decision making under certainty and of some of the more important tools for applying these developments. The book does not address decision making under risk and uncertainty. These topics are missing, not because they are uninteresting or transparently useless, but because they are so important that they merit a separate treatment. Moreover, Anderson, Dillon, and Hardaker (1977) have already provided a good book-length treatment.

The introduction surveys in a relatively succinct manner the chronology of production economics as seen through the eyes of an agricultural economist whose main interest in these tools and the theory behind them has been in applying them to agricultural problems. Thus, the book relies quite heavily on examples and intuition cast in an agricultural context.
Preface

Chapter 1 examines the properties of the single-output production function. In writing the book, much thought was given to whether the discussion should dispense with the production function from the start and rely solely on multioutput representations of the technology. My decision to start with the production function is based primarily on two reasons: First and foremost, my experience as a graduate instructor has shown me that, for whatever reason, students grasp the notion of a production function more readily than they do the more general concept of a technology; second, starting from the production function allows the theory and results developed in the first two chapters to be extended in a straightforward manner to the consumer case simply by changing definitions and terminology. But multioutput technologies are not ignored completely: They form the subject matter of Chapter 7.

The second and third chapters deal with the cost-minimizing decision. This is really the core of the book since most of the analytical tools used in other sections are developed at length in these chapters. The decision to develop the tools and the analytical concepts in terms of the cost function is largely based on a prior pedagogical accident: That was the way that I learned them. It is not intended to cast the cost minimization paradigm as inherently more important than the profit maximization framework; indeed, it is obvious that the former is always a special case of the latter.

Chapter 4 moves on to profit functions and is considerably less detailed in terms of mathematical exposition than the second and third chapters. An attempt has been made in this chapter to use as many heuristic arguments as possible without deviating too much from the straight and narrow of mathematical formalism. Perhaps it is well to point out here that the book relies heavily on mathematical arguments. The reader will encounter relatively few pages where he or she is not obligated to read at least one equation. Although this book and the course it emerged from are not and were never intended to be courses in mathematical economics, the use of so much mathematics is inevitable in a book such as this. But the reader can rest assured that with some distinctly minor exceptions, the arguments in this book only require a sound understanding of the differential and integral calculus.

Chapter 5 is devoted to an analysis of functional forms and aggregation. The decision to include half a chapter on functional forms in a book on applied production economics is obvious and needs little explanation even to the uninitiated. But the emphasis on aggregation may seem a little less obvious. I included this treatment because experience has taught me that only rarely does one encounter data sets that do not require aggregation across firms or across industries. Thus, I felt that the reader should
be well acquainted with the aggregation problem. Chapter 6 is devoted to different methods of measuring technical change. The chapter splits naturally into two parts: The first views technical change continuously and concentrates on the representation of technical change in both primal and dual functions; the second concentrates on the more discrete approach of using index numbers to measure technical change. As stated, Chapter 7 is devoted to multioutput representations of the technology.

During the years that I have been writing and rewriting this book, I have had many professional and personal experiences that have contributed greatly to the final product. I would like to take a few lines here at the reader’s expense to thank some of the people involved. Strangely enough, the two people I want to thank first taught me virtually nothing about production economics. Although Richard Just and Andy Schmitz may have taught me nothing about production economics, they taught me much about economics, economic research, and being an economist. Without their help and advice in the formative stages of my professional career, this or any other professional endeavor would not have been possible. Over the years, a number of professional colleagues and former students have either read parts of the manuscript or helped me to understand many of the concepts used in the manuscript by engaging in joint research and just plain conversation: Ted McConnell, Ramon Lopez, Rulon Pope, John Antle, Rolf Färe, Mike Weiss, Ana Aizcorbe, Tom Lutton, Bruce Gardner, Darrell Hueth, Ivar Strand, Ian Hardie, Utpal Vasanvada, Hyunok Lee, and Eldon Ball all fall in this category. I especially want to thank Arne Hallam, however, for reading the entire manuscript several times. Special thanks also go to Virgil Norton, who first convinced me to come to the University of Maryland and who, as chairman of my department, provided me with the ideal research support for a new assistant professor. Finally, I want to thank Virginia Smith, Ginger Kuykendall, Kathy Banvard, and Saroj Bhandari for typing and retyping bits and pieces of the manuscript.
Notation

\( V(y) \) input requirement set
\( x \) input vector
\( y \) output vector
\( w \) input price vector
\( p \) output price vector
\( \nabla(y) \) lower boundary of \( V(y) \)
\( c(w,y) \) cost function
\( \Pi(p,w) \) profit function
\( R(p,w) \) revenue function
\( \sigma \) elasticity of substitution
\( \sigma_{ij} \) Allen elasticity
\( \sigma^M_{ij} \) Morishima elasticity
\( \sigma^T_{ij} \) shadow elasticity
\( \epsilon_{ij} \) demand elasticity, cost minimizing
\( \epsilon_{ij}(p,w) \) demand elasticity, profit maximizing
\( T \) production possibilities set
\( Y(x) \) producible output set
\( R(y) \) factor price frontier
\( V^*(y) \) implicit input requirement set
\( c^*(w,y) \) implicit input cost function
\( \Pi(q, \theta) \) restricted profit function
\( I \) set of input indexes
\( \mathcal{I} \) partition of \( I \)