1

Forces and fields

In daily life we see the activity of forces all around us. The force of gravity guides the planets in their motion and raises the ocean tides. Electrical forces display themselves in thunderstorms. Mechanical forces drive our machines and our own bodies. Everywhere we look, matter is subjected to forces of some sort, arising from a multitude of agencies.

The study of these forces of nature is the job of the physicist, who wants to understand how the forces arise, what effects they have and whether any relationship exists between the different sorts of forces. Today it is fashionable to speculate that all forces have a common origin in a single 'superforce' that is explicitly revealed only at ultra-high energies.

Real progress towards these goals can only be made by appealing to quantum physics. In this chapter, however, we shall only be concerned with classical ideas of forces, as they apply in the macroscopic world. In later chapters these elementary concepts will be extended to the quantum microcosmos.

1.1 *Action and motion*

'In nature things move violently to their place
and calmly in their place'
(Francis Bacon, 1561–1626)

Why do things happen? For centuries this most fundamental of all questions about the physical world has lain at the foundation of science. An explanation for events must rest on a correct understanding of the natural condition of objects. The world is full of objects – people, planets, clouds, atoms, flowers – and full of motion. Things happen when moving objects act collectively. How do objects *know* about each other? How do they respond to the presence and activities of other objects?

Before the seventeenth century the natural state of all objects on Earth was thought to be the state of *rest* (see the quotation above). In daily experience moving objects soon return to rest if they cease to be powered by something. Switch off the engine of a motor car and it grinds to a halt in a few hundred metres.

People did not ask: 'Why does a body remain at rest?' Instead they asked: 'Why does a body move?' The Greek philosopher Aristotle (384–322 BC) taught that all motion requires a *cause*, an agency to produce it. For example, the flight of an arrow is sustained by a vortex of air surrounding it. Without an agency to sustain it, all motion naturally ceases. For Aristotle, any activity implied *action*.

Aristotle's ideas were a failure. A proper understanding of motion, action and activity had to await the epochal work of Isaac Newton (1642–1727). Central to Newton's concept of motion is that the state of rest is only one of many possible natural states for an object. A body can also *move uniformly* without any external agency to sustain it. That is, it will continue to move in a straight path with undiminished speed unless something operates to change it.

The idea that uniform motion does not require any explanation is intimately bound up with the notion that such motion is merely *relative* to whoever observes it. The relativity of motion will be further discussed in Section 1.5, but for now we simply note that the bowl of soup which is at rest on the table before the airplane passenger is travelling at several hundred kilometres per hour over the head of the city dweller below. The 'natural' state of rest for the passenger is a 'natural' state of steady motion for the city dweller.

If uniform, undiminished motion is the natural order of things, why do we need engines for motor cars? According to Newton's laws the motion as such of the freewheeling car does not require explanation. Instead we must explain why it slows down. The explanation is readily available. Friction and air resistance sap the energy from the moving car. In outer space, where there is no air resistance, objects continue to move without propulsion. If this were not so the unpowered Earth would slow in its orbit and fall into the sun.

Although uniform motion is natural and needs no explanation, *changes* in motion require the action of some external agency. Because the state of uniform motion is regarded as natural, we say that when a body is disturbed from this state it is being *forced*. The agencies which produce forced motion are called forces. It is the action of forces which enriches the activity of our universe, and which enables different parts of the world to be aware of each other's existence. Without forces, nothing could act on or influence anything else, and all the matter in the universe would disintegrate into its elementary constituents, each subatomic particle moving independently of all the others.

The effect of a force on a material body is to bring about an acceleration. This is described by Newton's second law of motion.

The familiar statement of the second law is that the action of a force F produces on a body an acceleration a in proportion to the magnitude of F and in the same direction as F acts. A given force will accelerate a heavy body less than a light one, and this is taken into account by introducing the idea of *inertia*. The inertia of a body is a measure of its resistance to changes in motion, and roughly speaking one can think of more massive bodies as having a greater inertia than less massive ones. (It is easier to push a bicycle than a car.) If we denote the mass by m then the second law states

$$F = ma \tag{1.1}$$

or in words, force = mass × acceleration.

To determine how a body responds to a given force F, which may be varying from time to time and place to place in both magnitude and direction, it is necessary to solve Equation (1.1) for the *position* of the body. This can be done because the acceleration a is the rate at which the velocity of the body changes with time and the velocity is, in turn, the rate of change of displacement of the body from some fixed place. Solving Equation (1.1) therefore amounts to determining how the *location* of the body changes from time to time, i.e. it determines the *trajectory* or *orbit* which the body follows.

All forces have both a magnitude and a direction. Quantities that can be directed are called *vectors*; we shall meet several examples of vectors in this book. They can be conveniently denoted by arrows. Vectors can be added, but the rules of addition are more complicated than those for numbers. For example, if we add two vectors of equal magnitude but opposite direction they cancel each other completely (see Fig. 1.1). On the other hand, if they are parallel, they reinforce to produce a vector in the same direction but with twice the magnitude. Intermediate cases produce vectors with intermediate magnitudes *and* directions.

Other examples of vectors are velocity, momentum and acceleration. In contrast, temperature and energy have magnitudes at each place, but are not directed. They are not vectors. Newton's second law relates force to acceleration, so it is a vector equation. It gives information not only about the magnitude of the acceleration that a given force produces, but also about its direction – the body is accelerated along the line of action of the force.

When a force acts on a body and it accelerates, its *momentum* changes. Momentum is a familiar property in daily life. Heavy, fast-moving bodies carry a lot of momentum; light, slow bodies very little. Physicists make this concept precise by defining momentum to be the product of mass (m) and velocity (v): momentum $= mv$. Now acceleration is simply the *rate* at which velocity changes, so if the mass m remains constant, Equation (1.1) can be written

$$\text{force} = \text{rate of change of momentum.} \quad (1.2)$$

Because velocity is a vector, a body can accelerate by changing either the magnitude of its velocity, i.e. its *speed*, or its direction, or both. When a football is kicked from rest it accelerates by changing its speed only. When a bullet ricochets off a target it changes its direction and may also lose speed. If a spacecraft orbits around the Earth in a circle, it does not alter its speed, but it still accelerates continually by changing the *direction* of its motion. All these types of motion alter the momentum of the body and all require a force to produce them.

If no force acts on a body its momentum will not change: it is *conserved*. Momentum may be added like any other vector. The total momentum of a system is the sum of the momenta of its constituent parts. If a system is isolated from external forces, its total momentum will not change, but it may still experience internal forces which can rearrange the total momentum among the compo-

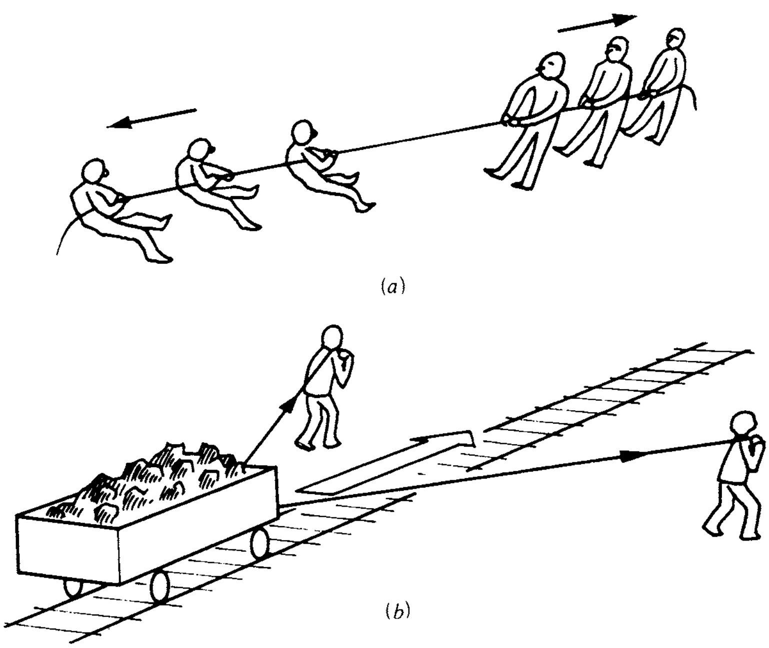


Fig.1.1. Vectors. Quantities such as force and acceleration have both direction and magnitude: they are called vectors. When vectors are added, account must be taken not only of how large they are, but also which way they point. In the example shown in (a) the directions of the forces are opposed and cancel each other, even though their individual magnitudes may be large. In (b) obliquely directed forces combine to give a net force in a direction between them with a strength somewhat less than their combined magnitude.

nents. The law of conservation of momentum tells us that the vector sum of all this momentum cannot increase or decrease.

A good example is provided by the gun. At rest, it has zero momentum, so the combined system gun + bullet must also have zero momentum after the bullet has been fired. The force which propels the bullet is purely internal to the system and reacts with an equal force back on the gun. The outcome is a *recoil*. The momentum carried by the recoiling gun is equal in size and opposite in direction to that of the bullet. Because the mass of the bullet is so much smaller than that of the gun, the *speed* of recoil is much less than the speed of the bullet. Another example is given in Fig. 1.2.

A related concept is angular momentum which concerns the rotation of bodies. The angular momentum of a point mass m rotating with velocity v in a circle of radius r, is mvr. Newton's laws require that, in the absence of external forces, angular momentum is also conserved. A good example of this is the spinning ice skater who, by contracting her arms and thereby concentrating her mass closer to the axis of rotation, conserves angular momentum by increasing her rotation rate.

Angular momentum is a vector, and is described by drawing an arrow along the rotation axis. There is a convention about the direction of the arrow, best remembered by imagining the rotation of the hands of a clock. The arrow describing the hands' angular momentum points *into* the front of the clock (see Fig. 1.3).

There is an important distinction, however, between vectors associated with angular momentum and those associated with linear momentum. The distinction concerns the properties of these vectors under reflections. If Fig. 1.3 is viewed in a mirror, the direction of rotation appears reversed (i.e. anticlockwise) and the arrow now appears to point the wrong way according to the convention mentioned above. More precisely, one can examine whether a vector changes sign when referred to a cartesian coordinate system (x,y,z), and the coordinates are reflected in the origin to $(-x,-y,-z)$.

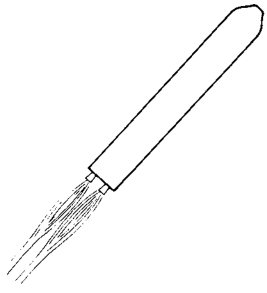


Fig. 1.2. The motion of a rocket is explained by the law of conservation of momentum. The exhaust gases ejected backwards propel the craft forwards so that the total momentum of the system is unchanged.

Vectors which change sign under reflection are called *polar vectors*. Some examples are linear momentum, acceleration and electric field. Vectors like angular momentum and magnetic field do not change sign under reflection. These are called *axial* vectors. In Chapter 5 we shall see that the distinction between axial and polar vectors is central to understanding certain particle transmutations.

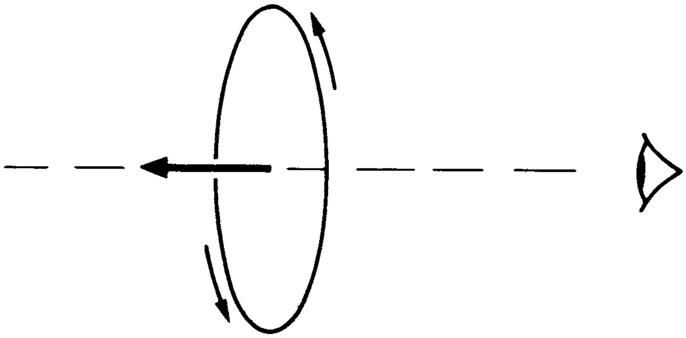
Another very important conservation law concerns *energy*. There are many different forms of energy; for example heat, light, electricity, kinetic energy, gravitational potential energy. Often energy is converted from one form into another. Consider, for example, the propulsion of an arrow. The archer supplies energy to stretch the bow string, where it is temporarily stored as potential energy. When the string is released the energy is rapidly converted into the motion of the arrow. When the arrow strikes the target, the motion is arrested and the energy converted into heat, sound and the work done parting the material of the target. Except in certain exotic gravitational processes (and neglecting quantum effects for now), energy is always found to be conserved during such conversions.

So far nothing has been said about the nature of the forces which act on matter. In daily life we encounter many different sorts of forces. Below are listed some familiar examples:

Pull of a rope	Lightning strike
Blow of a hammer	Raising of the tides
Gravity	TNT explosion
Friction	Magnetism
Air resistance	Wind pressure
Human muscles	Hydraulic pressure

Notice that some of these forces, such as the blow of a hammer, act by direct contact with the body concerned while others, such as gravity, reach out across empty space.

Fig. 1.3. The vector associated with rotation is labelled by an arrow which points in the direction of insertion of a right-handed corkscrew.



Remarkably all the forces listed can be shown to arise as the result of just two fundamental forces – gravity and electromagnetism. We shall look at each of these in turn.

1.2 *Gravitation*

The force of gravity is familiar to us because it keeps our feet on the ground. It is, however, by far the weakest of all forces. The reason it is so conspicuous in daily life is due to the proximity of the vast bulk of the Earth. However, gravitation exists between all material bodies. With sensitive equipment the attraction between small objects can be measured in the laboratory.

Newton was the first person to produce a workable theory of gravitation. He realized that it is a universal force, acting at a distance between the stars and planets as well as at the Earth's surface. The strength of the force between two idealized point masses is given by Newton's famous inverse square law

$$F = GM_1M_2/r^2 \tag{1.3}$$

where M_1 and M_2 are the masses of the two bodies, r is their separation and G is a universal constant with the value $6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$. Newton's force of gravitation is always attractive.

Using this theory of gravitation together with his laws of motion, Newton was able to explain the sizes and shapes of the planetary orbits in the solar system, and the ocean tides due to the gravity of the moon and sun. Today, the successful application of his theory to the motion of planets, asteroids and spacecraft has reached a very high level of precision.

Although Newton's theory of gravitation is adequate for most technological and astronomical purposes, it is known to fail when gravitational forces become very strong, or the bodies involved move at speeds approaching that of light. These circumstances require a more complete theory of gravitation. In 1915 German-born genius Albert Einstein (1879–1955) published such a theory, called the general theory of relativity. The key element in Einstein's theory is that gravitation is not treated as a force at all, but as a curvature or distortion in the geometry of space and time. This is a difficult and abstract idea, but it implies that, crudely speaking, the Earth orbits the sun in a curved path not because the sun forces it from straight-line motion, but because the sun produces a warp in space–time in its vicinity; and the Earth then travels the straightest available route through the warped geometry.

In spite of its fundamentally different conceptual basis, Einstein's theory reduces to Newton's under most circumstances of interest.

Where major differences arise is in the collapse of stars and other objects. The general theory of relativity predicts that a totally imploded star will produce severe distortions of space and time, leading to objects known as black holes. In a black hole even light is trapped in a space–time prison.

Another crucial difference between the two theories is that, in general relativity, it is not only material bodies that gravitate. Energy, pressure and stress forces also produce gravity. Moreover, whereas in Newton's theory gravitational forces act instantaneously across space, Einstein's theory describes gravitational action in terms of a field (see p. 9) in which disturbances propagate no faster than the speed of light. One consequence of this is the prediction of gravitational waves, acting like ripples in the gravitational field. The detection of these waves is currently a major challenge to experimental physicists.

1.3 *Electricity and magnetism*

Electric and magnetic forces are easily demonstrated and were known in ancient civilizations. Laboratory experiment established that electrically charged bodies can both attract and repel each other. This led to the realization that electric charge can be either negative or positive, with the rule that like charges repel and unlike charges attract. The strength of the force diminishes with the separation between charges in exactly the same way as Newton's law of gravitation, i.e. an inverse square law. Thus, for point charges of magnitude e_1 and e_2, the force is

$$F = Ke_1e_2/r^2. \tag{1.4}$$

The constant K, analogous to Newton's constant G, has a value which depends on the system of units used to measure charge. In SI units it is usual to write $K = 1/4\pi\varepsilon_0$, where ε_0 is called the permittivity of free space and has a value of 8.85×10^{-12}.

A fundamental property of electricity is expressed by the law of conservation of charge, which states that in an isolated system the total quantity of positive charge minus the total quantity of negative charge always remains the same. Charge can never be created or destroyed. This means that when electricity is produced in an electrically neutral system, any positive charge that is created is always accompanied by an equal quantity of negative charge.

All ordinary matter contains electric charges, but the electrical properties of different materials vary widely. Some substances produce electricity when merely rubbed. The Ancient Greeks noticed that amber rubbed with fur would attract small objects, and our word 'electricity' derives from the Greek word for amber (*electron*).

Certain materials permit the free flow of electricity through them. These we call conductors. In contrast, insulators block the flow of electricity.

Most people's initial encounter with magnetism is the compass, which is attracted by the Earth's magnetism to align north–south. This phenomenon has led to the use of the terminology 'north' and 'south' magnetic poles to describe the magnetic equivalent of electric charge. As with electricity, like poles repel and unlike poles attract with a force that obeys an inverse square law.

Electric and magnetic forces were extensively studied in the early part of the nineteenth century and investigators sought a more physical explanation of how it is that a force can act across empty space between two magnets or two electric charges. The basic problem is illustrated by Fig. 1.4(*d*). If two metal balls are suspended side by side they will hang vertically, unaware of each other's existence. Suppose a compressed spring is now introduced between the balls and allowed to expand. The system will take up a new configuration with the balls hanging obliquely. The cause of the disturbance is visible: the spring is in contact with each ball, and provides a continuous communication between them through its coils. The force of the spring pushes the balls apart. Now suppose that instead of using a spring the balls are given some electric charge of like sign. The effect is the same, the balls repel each other and hang at an angle to the vertical, only in this case there is no visible means of support. What agency is responsible for communicating the electric interaction?

An answer to this question is to suppose that each electric charge produces in its vicinity a so-called electric *field*. The field, which extends outwards through the space surrounding the charge, is invisible. Its presence is inferred from the action that it has on other electric charges which encounter it. In this way, action-at-a-distance between charges becomes action-by-contact between one charge and the field of the other.

The field concept was introduced by Michael Faraday (1791–1867) and extensively developed by James Clerk Maxwell (1831–79). The idea can be extended to magnetic (and gravitational) forces too. The electric charge or magnetic pole is regarded as the *source* of the field. The strength of the field progressively diminishes with distance from the source. Far from the source, where the field is weak, the force on another electric charge or magnetic pole is correspondingly weak too, in accordance with the inverse square law.

Textbooks often depict the shapes of electric and magnetic fields by showing patterns of field lines, or lines of force as they are known. Some examples of electric lines of force are given in Fig. 1.5. Note that lines only begin or end on charges: the field cannot abruptly disappear in empty space. The fields depicted here actually extend to infinity, so strictly speaking there is apparently no distance beyond which a test charge would not feel a force. For this reason electric fields are referred to as *long-ranged*, a property shared by magnetism and gravitation. If this were not so, and the fields only extended for a finite distance, the theory would contain a fundamental scale of length. The absence of such a scale implies certain important symmetry properties for these forces, which turn

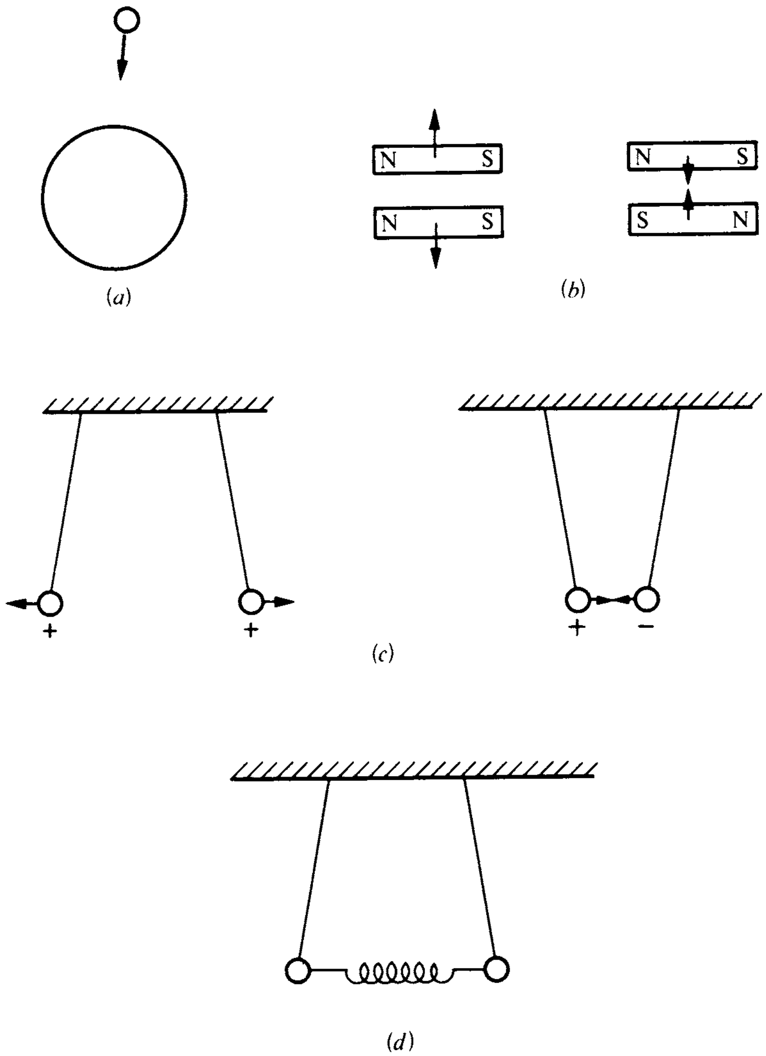


Fig. 1.4. Action-at-a-distance. (*a*) Gravity reaches out across empty space to pull on all other matter with a force that diminishes with distance in a mathematically precise fashion (inverse square law). (*b*) Magnetism both attracts and repels other magnetic substances even though they are not in physical contact. Magnetic poles come in two types, north and south. Their mutual force also diminishes with separation. (*c*) Electricity provides a third example of action-at-a-distance. The like charges on the suspended balls force them apart, and the unlike charges draw them together, with a force that yet again diminishes with separation in an inverse square law. (*d*) A similar effect can be produced by inserting a spring between the balls, though in this case the cause of their repulsion is visible.