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Introduction to Linear and Convex Programming

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PREFACE

'The student gets good *profit* from his plough-team when early spring comes round - the frame of his plough is a handful of *pens*!'

(Celtic proverb)

This book has grown from courses given at Monash University since 1981 and is suitable as a textbook for third year undergraduate students of mathematics as well as students of economics, operations research, engineering, etc.. A preliminary version of the first two chapters was used for a second year course on linear programming (although topics such as equilibrium and stability are here discussed in the fourth chapter).

The book has two main objectives. One is to introduce optimization, a significant modern branch of applied mathematics; the other, more subtle, is to carefully study linear algebra, euclidean space geometry and some analysis in an applied context, preliminary to a study of functional analysis. The courses at Monash University are designed to help students of applied mathematics and pure mathematics appreciate the need for both points of view.

In the words of the late Richard Bellman, 'The only sensible way to handle multidimensional matters is by the use of vector-matrix notation.' Chapter 1 deals with the linear algebra and geometry needed for understanding and solving linear programming problems, using the 'sensible way'. Chapter 2 studies linear programming, its applied context, duality theory and the simplex algorithm. The simplex method, which is shown to *always* solve LP problems, makes use of the lexicographic ordering of euclidean space. Some attention is paid to computer implementation of the algorithm, by using a revised simplex method (a fine example of the power of matrix notation).

Chapter 3 contains very aesthetically satisfying, useful mathematics. Some elementary results from topology (of euclidean space) are needed and proofs contain arguments of a kind familiar to students of analysis (as in Binmore (1982)). The results, while often not easy to prove, have natural geometric appeal. The approach owes much to Fenchel (1949), Rockafellar (1970) and Ponstein (1980). In chapter 4 students are brought close to present day developments in convex programming theory. The Fenchel transforms introduced in the third chapter provide a key to the duality theory, once the idea of perturbation space is introduced. Students at Monash University expressed appreciation, and perhaps surprise, at how well everything fits in to place in chapter 4, but not without some hard work!

It is important for understanding that worked examples be carefully studied (using pen, or even pencil, and paper) and that many of the exercises be attempted. Solutions and comments on the exercises are provided. Some of the tools introduced in the book, such as subgradients, are useful in other fields like numerical analysis and the geometry of Banach Spaces. The notation := is used throughout the book to mean 'is by definition'.

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Neil Cameron
Frontmatter
More information

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Neil Cameron,
Monash University,
February, 1985.

