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Syllogistic consequence

The first sentence of the *Prior Analytics* states that the subject of inquiry is proof. However, Aristotle first presents his theory of the syllogism because, he says, it is more general: every proof is a syllogism, but not every syllogism is a proof (*An. Pr.* 25b28–31). Aristotle defines a syllogism as a '*logos* in which, certain things being posited, something other than what is posited follows of necessity from their being so' (*An. Pr.* 24b18ff).[1] What is it to follow of necessity? And how does Aristotle show that, given certain premisses, a conclusion follows of necessity? This chapter provides an introduction to Aristotle's logical programme.

The modern logician works with two notions of logical consequence. One is semantic: the logician provides an analysis of what it is for an arbitrary sentence to be true in a model. Then a sentence P is said to be a semantic consequence of a set of sentences X if P is true in every model in which all the members of X are true.[2] This semantic definition of consequence provides an analysis of what we mean by saying that P is a logical consequence of X if *whenever* all the members of X are true, P *must* be true. That is, it provides an analysis of what it is for a sentence to follow of necessity from other sentences. The other notion of consequence is syntactic. The logician specifies effective rules for manipulating symbols of a language and P is said to be a syntactic consequence of a set of sentences X if one can move from X to P using only the specified rules. Of course, the syntactic rules are chosen with the intention that the rules will at least preserve and if possible capture the relation of semantic consequence. In modern logic one proves formal inferences sound with respect to a semantics. From a modern

[1] The usual translations of *logos* in this context, e.g. 'discourse', 'argument', are not adequate. 'Discourse' may suggest dialogue or conversation which should not be present, 'argument' may suggest argumentative force which a syllogism need not possess (see Chapter 3 below). The problem of translating *logos* is of course not new. Says Goethe's Faust: 'I feel that I must open the fundamental text: must try, with honest feeling, to set down in my own beloved German that sacred original. It is written: "In the beginning was the Word!" Already I have to stop! Who will help me on? It's impossible to put such trust in the Word! I must translate some other way if I am truly enlightened by the spirit.'

[2] See Tarski, 'On the concept of logical consequence'.

perspective, it is a soundness proof which justifies a particular syntactic inference.[3]

However, it has become too easy to assume that a syntactic inference *must* be justified by some form of semantical soundness proof. This is because logicians have tended to treat formal systems as uninterpreted, as a safeguard against theoretical assumptions remaining hidden in the underlying logic.[4] The syntactical relation of formal deducibility is then defined as a relation between uninterpreted symbols of a formal language. The definition of such a relation depends upon an antecedent analysis of logical consequence, such as Tarski's, but, taken strictly as a relation among uninterpreted symbols, it is not a *consequence* relation at all. A syntactical relation, however, need not be restricted to uninterpreted symbols of a formal language. Of course, one must be able to determine whether a finite string of symbols is a formal derivation without recourse to their interpretation. One may nevertheless regard the rules of inference and deducibility relation as holding among interpreted sentences. In so far as the syntactic relation is genuinely one of consequence, it must contain a semantic ingredient.

To understand Aristotle's logical programme, it is crucial to distinguish a syntactic relation from a relation between uninterpreted symbols. For if one conflates 'syntactic' with 'uninterpreted', it seems one must provide a semantic analysis of consequence which the syntactic relation is supposed to capture. Aristotle does not offer a definition of 'following from necessity' and then show that the syllogisms are true to it. Rather he begins by presenting a few obviously valid inferences and invites one to agree that these are cases in which the conclusion follows of necessity from the premisses. The syllogisms of the first figure –

$$\frac{Aab\ Abc}{Aac} \qquad \frac{Aab\ Ebc}{Eac} \qquad \frac{Iab\ Abc}{Iac} \qquad \frac{Iab\ Ebc}{Oac}$$

– are said to be perfect (*An. Pr.* 25b32ff). A syllogism is *perfect* if it needs nothing other than what is stated to make evident what necessarily follows (*An. Pr.* 24b22–25). Hence to establish that the conclusion of a perfect syllogism follows from the premisses, one should need to do no more than state the syllogism itself. For first figure syllogisms, this is virtually all that Aristotle does (cf. *An. Pr.* 25b37–26a2; 26a23–27).

[3] See Dummett, 'The justification of deduction'.

[4] For example Kleene says, 'First the formal system itself must be described and investigated by finitary methods and *without making use of an interpretation of the system*' (my emphasis). *Introduction to Metamathematics*, p. 69.

At *Prior Analytics* 26b29 he simply states that it is evident that the first figure syllogisms are perfect. No argument is given for their validity. For if the syllogisms are perfect, no argument need be given.

Aristotle also introduces three rules of conversion:

From *Eba* infer *Eab*
From *Aba* infer *Iab*
From *Iba* infer *Iab*

He presents them as follows:

> 'In universal belonging it is necessary that the terms of the negative premiss should be convertible, e.g. if no pleasure is good, then no good will be pleasure; the terms of the affirmative must be convertible, not however universally, but in part [i.e. to a particular proposition], e.g. if every pleasure is good some good must be pleasure; in particular belonging, the affirmative must convert in part (for if some pleasure is good, then some good will be pleasure); but the negative need not convert, for if some animal is not a man, it does not follow that some man is not an animal.' (*An. Pr.* 25a5–13)
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> 'First then, take a universal negative with the premiss *ab* [*Eba*]. If *a* belongs to no *b*, neither will *b* belong to any *a*. For if *b* belonged to some *a*, for example to *c*, it will not be true that *a* belongs to no *b*; for *c* is a *b*. But if *a* belongs to every *b*, then *b* will belong to some *a*. For if *b* belonged to no *a*, neither will *a* belong to any *b*: but it was assumed that *a* belongs to all *b*. Similarly too if the premiss is particular. For if *a* belongs to some *b*, then necessarily *b* belongs to some *a*: for if *b* belonged to no *a*, neither would *a* belong to any *b*. But if *a* does not belong to some *b*, it is not necessary that *b* does not belong to some *a*, e.g. if *b* is animal and *a* is man. Man does not belong to all animal, but animal belongs to all man.' (*An. Pr.* 25a14–26)

The point of Aristotle's argument is to get one to recognize these inferences not merely as valid, but as obviously valid. The passage 25a5–13 illustrates the three rules of conversion using the terms 'pleasure' and 'good'. The intention is that one simply see that the rules of conversion are true for these examples and that the examples are illustrative of valid rules. It would, of course, be a mistake to interpret 25a5–13 as offering a proof of the rules of conversion, for invalid inference patterns may have particular instances in which the premisses and conclusion are true. Consider, for example, 'if some pleasure is not good, some good will not be pleasure, therefore the terms of the particular *Oca* must convert'. Even if some good is not pleasurable and some pleasure is not good, this does not justify the convertibility of the particular negative premiss in general. One does not know that one has taken arbitrary terms, terms that are genuinely illustrative of a valid inference pattern, unless one knows that the inference pattern they illustrate is valid. But one *can* recognize Aristotle's examples as instances of valid inferences and that is because the inferences they illustrate are obviously valid.

Similarly, the argument which follows in 25a14–26 should not be viewed as a proof of the rules of conversion from principles which are logically or epistemically prior. In this passage Aristotle introduces term variables which transcend the problem of knowing that particular terms (e.g. 'good', 'pleasure') are genuinely arbitrary and illustrative of a valid inference. He also employs both *ekthesis* and argument by *reductio ad absurdum*. *Ekthesis* occurs in the step:

> 'For if *b* belonged to some *a*, for example to *c*, it will not be true that *a* belongs to no *b*; for *c* is a *b*.' (*An. Pr.* 25a16)

In my opinion, *ekthesis* is similar to the use of free variables in modern systems of natural deduction. Having assumed that some *a*'s are *b*, we are allowed to select an arbitrary particular instance of *a*, which is *b*. This corresponds to existential instantiation in natural deduction. So *c* should not be seen as another term variable like *a* and *b*, but as an arbitrary instance of an *a*. This view is not uncontentious:[5] others believe that *c* should be interpreted as a term variable having as an extension those *a*'s that are *b*. Whichever view of *ekthesis* is correct, the important point for the thesis I am advancing is that one not take Aristotle to be giving a proof of the rules of conversion according to any logically or epistemologically prior technique. The argument is designed solely to display the obviousness of the validity of the rules of conversion. Consider, by way of analogy, the modern rule of and-introduction: 'From *P* and *Q*, infer *P-and-Q*.' One would expect that anyone who understood conjunction would simply see that this inference is valid. No proof of validity could employ rules more evidently valid than this. Still, to make the obviousness of the inference apparent, one might argue 'Suppose *P* and *Q* but not *P-and-Q*. If not *P-and-Q* then either not *P* or not *Q*, but that is absurd, since one has *P* and *Q*.'

[5] Cf. e.g. Lukasiewicz, *Aristotle's Syllogistic From the Standpoint of Modern Formal Logic*, pp. 59–67; Patzig, *Aristotle's Theory of the Syllogism*, pp. 156–68; Kneale, *The Development of Logic*, p. 77.

This is not a proof of and-introduction from logically prior rules or principles: no such proof is needed. With an apparently obvious inference, a doubt may remain whether something has been overlooked, whether one has fully understood the inference. The direct derivation of an absurdity from supposing the inference invalid reveals that the appearance of obviousness is genuine.

A syllogism is *imperfect* if it needs additional propositions set out, which are necessary consequences of the premisses, in order to make it evident that the conclusion follows from the premisses (*An. Pr.* 24b24). Patzig has noted that this definition presupposes that all imperfect syllogisms can be perfected.[6] Aristotle does not admit a category of unobvious syllogisms *per se*: syllogisms are divided exhaustively into those that are obvious and those that can be made obvious. The perfection of an imperfect syllogism '*P*, *Q* so *R*' consists in showing how one can move from the premisses *P* and *Q* to the conclusion *R* using the rules of conversion and first figure inferences (*An. Pr.* A5, 6). An example is the perfection of *Cesare*, '*Enm, Aom* so *Eon*', in the second figure:

> Since *Enm*, by conversion, *Emn*; but since *Aom* one can form the perfect first figure syllogism *Celarent* '*Emn, Aom* so *Eon*' (cf. *An. Pr.* 27a3ff).

Aristotle's strategy is to isolate a handful of obviously valid inferences and justify the remaining inferences by showing that they are redundant: one can move from premisses to conclusion without them. In the three figures Aristotle considers 48 possible pairs of premisses. Aside from the perfect first figure syllogisms, he is able to eliminate by counterexample all but ten other premiss-pairs as having no syllogistic consequences.[7] The remaining ten syllogisms can be perfected: they can, in Aristotle's words, be *reduced* to first figure syllogisms (*An. Pr.* 29b1).

Such a strategy demands a flexible conception of the means of perfection. Most notably, the moods *Baroco* ('*Acb, Oab* so *Oac*') and *Bocardo* ('*Obc, Aba* so *Oac*') are perfected by *reductio ad absurdum* arguments (*An. Pr.* 27a36, 28b17).[8] The problem with these syllogisms

[6] Patzig, *Aristotle's Theory of the Syllogism*, p. 45.
[7] See Chapter 4.
[8] One must be careful to distinguish the reduction of one syllogism to another, which uses a *reductio ad absurdum* argument, from a *per impossibile* syllogism. See e.g. Kneale, *The Development of Logic*, pp. 76–9; Patzig, *Aristotle's Theory of the Syllogism*, pp. 144–56. *Per impossibile* syllogisms are discussed in Chapter 3.

is that since the particular negative premiss does not convert, the only possible conversion that can be applied is one from the universal affirmative premiss to a particular affirmative premiss. For example, with *Baroco*, the only valid conversion possible is from *Acb* to *Ibc*. This leaves two particular premisses – *Ibc*, *Oab* – and there is no perfect inference with two particular premisses.[9] Aristotle is thus forced to abandon the direct method of perfection he has been using. To derive the conclusion *Oac*, he assumes its contradictory *Aac* and then infers, by the perfect first figure syllogism *Barbara*, an impossible conclusion:

> Suppose *Aac*, then since *Acb* it follows that *Aab*; but that is impossible since *Oab*; therefore *Oac*.

The claim that for any imperfect syllogism '*P*, *Q* so *R*' one can prove *R* from *P* and *Q* using only perfect inferences must therefore be treated with caution: it is true only if we are willing to countenance certain deviant means of perfection that are needed to make the claim true. The value of the doctrine of perfection – that all syllogisms are exhaustively partitioned into those that are perfect and those that can be made perfect – is that Aristotle is able to present a coherent logical theory without giving an analysis of the concept of logical consequence. For perfect syllogisms one can simply point to their validity; for imperfect syllogisms one justifies them by showing how they can be perfected.

The debate that has ensued since Aristotle's time over the obviousness of perfect syllogisms has focused on two related issues: (1) What is it about the perfect syllogisms that makes their validity evident? (2) What is it about the imperfect syllogisms that makes them less evidently valid than perfect syllogisms?

Kneale has suggested that in first figure syllogisms the terms are arranged so that the transitivity of the relations 'belongs to' and 'is predicated of' is evident.[10] Kneale notes that Aristotle presents two distinct formulations of a first figure syllogism. One formulation talks of one term being in another as in a whole (25b31); the other talks of one term being predicated of all of another (25b37ff).

> 'Whenever three terms are so related to one another that the last is in the middle as in a whole, and the middle is either in, or excluded

[9] Indeed there is no formally valid syllogistic inference at all with two particular premisses.
[10] Kneale, *The Development of Logic*, p. 73.

from, the first as in or from a whole, the extremes must be related by a perfect syllogism . . . If *a* is predicated of all *b* and *b* of all *c*, *a* must be predicated of all *c* . . .' (*An. Pr.* 25b31–39)

In these formulations, Aristotle reverses the order in which the terms are presented, thus preserving the obviousness of the transitivity of each relation. If this analysis of why Aristotle called the first figure syllogisms perfect is correct, then, as Patzig has said, the debate over what makes perfect syllogisms perfect has occurred in a misleading context.[11] For the traditional formulation of a syllogistic premiss 'All *a*'s are *b*'s' rather than the Aristotelian '*b* is predicated of all *a*' or '*b* belongs to all *a*', in conjunction with the presentation of the syllogistic premisses in the same order as Aristotle presented them, destroys the very feature of the first figure inferences that is supposed to make them perfect.

What I should like to argue, however, is that these questions – of what it is that makes first figure syllogisms perfect and whether or not the second and third figure syllogisms are less obviously valid – though of interest in themselves, are irrelevant to the development of Aristotle's logical programme. All that is crucial to his programme is that there be agreement *that* the first figure syllogisms are obviously valid. One need not know why. If Aristotle was unable to provide an explicit analysis of the relation 'follows of necessity', and took it as primitive, he may equally well have been unable to articulate what it is to follow obviously of necessity. Further, Aristotle is far less committed to the unobviousness of the imperfect syllogisms than he is to the obviousness of the first figure syllogisms. For the first figure syllogisms form the basis of a logical programme that is carried out in the absence of an analysis of the concept of syllogistic consequence.

Because Aristotle did not offer an analysis of 'follows of necessity' there is an indeterminacy in the strength of this consequence relation. This is reflected in the fact that for any terms *a*, *b*, *c* 'Aab & $Abc \supset Aac$' will be true if and only if in every interpretation in which Aab and Abc are both true, Aac will be true. A similar situation holds for every valid syllogistic inference. Thus, for every syllogism, the syllogism will preserve truth for any substitution of terms in the language if and only if in every interpretation in which the premisses are true, the conclusion is true. It would be anachronistic to ascribe to Aristotle a modern conception of semantic consequence: the concept of a language

[11] Patzig, *Aristotle's Theory of the Syllogism*, pp. 57–61.

having various interpretations is too recent and hard-won a discovery. Rather, Aristotle is working with the presemantic idea of interpretation by replacement: a statement-form is seen to have various instances. One obtains an interpretation of a syllogistic formula by substituting specific terms, of the appropriate logical category, for the schematic letters. Every syllogistic inference is *valid under replacement* in that for every substitution of terms which makes the premisses true, the conclusion is true.[12] This, however, only sets a lower bound on the strength of the syllogistic consequence relation. One cannot recover the precise strength of the relation of following of necessity.

Why was Aristotle able to take 'follows of necessity' as a primitive notion? One trivial reason is that there is a sufficient variety of Greek common nouns. When Aristotle wishes to show, for example, that the rule of conversion 'From *Oac* infer *Oca*' is invalid, he uses the terms 'animal' and 'man' (*An. Pr.* 25a12, 25a22ff). Not all animals are men, but it does not follow that not all men are animals. To show the validity of the other rules of conversion, Aristotle used the terms 'pleasure' and 'good'. These terms are inappropriate to reveal the invalidity of 'From *Oac* infer *Oca*' because, arguably, some pleasure is not good and some good is not pleasurable. Imagine for a moment that all common nouns in Greek happened to be such that if *Oac* is true, then *Oca* is true. Aristotle would have had to resort to some form of semantical argument by interpretation if he were to establish the invalidity of this inference. A more substantial reason is that Aristotle is willing to expand the means of perfection. Suppose, for example, that Aristotle was not acquainted either with argument by *reductio ad absurdum* or with *ekthesis*. A problem would then arise with the perfection of *Baroco* or *Bocardo*, for, as we have seen, neither can be perfected in the normal way, by a series of conversions. Because he is willing to countenance deviant methods of perfection, Aristotle is able to take 'follows of necessity' as a primitive. For he is able, by hook or by crook, to reduce the unobvious syllogisms to the obvious; and the obvious he is content to leave unexplained.

Whatever the strength of the consequence relation, a syllogism is something that has structure as well as semantic force. Łukasiewicz and, following him, Patzig, have argued that the syllogism is not an inference from premisses to conclusion, but a conditional in which the premisses function as a conjunctive antecedent and the conclusion as a

[12] For a discussion of validity under replacement, see Michael Dummett, *Elements of Intuitionism*, pp. 218ff.

consequent.[13] For example, the syllogistic mood *Barbara* is treated not as an inference, but as a single sentence 'If *Aab* and *Abc* then *Aac*.' This interpretation has already been seriously discredited by Smiley and Corcoran,[14] but it is nevertheless worthwhile for the present inquiry to see what is wrong with it. First, the opening sentence of the *Prior Analytics* states that the scope of inquiry is proof (24a10) and one cannot make sense of the claim that a proof is a type of syllogism (25b28ff) if one treats a syllogism as a conditional. A proof is an argument, with definite structure, from several sentences functioning as premisses to a conclusion. It is not a single sentence. Second, Aristotle's distinction between direct and *per impossibile* syllogisms refers solely to the manner in which conclusions are derived. In a *per impossibile* syllogism Aristotle says that one supposes the contradictory of what one wishes to prove and then derives an admittedly false conclusion (*An. Pr.* 62b29–31; 41a23–24).[15] For example, to prove *Abd* one argues:

> Suppose *Obd*, then since ***Abc***, it follows that *Ocd*; but ***Acd***; therefore *Abd*.

(The premisses are in bold type.) Aristotle shows that the premisses of this *per impossibile* syllogism provide the premisses for a direct syllogism with the same conclusion:

$$\frac{Abc \quad Acd}{Abd}$$

Aristotle shows that any conclusion that can be derived by a direct syllogism can also be derived, from the same premisses, by a *per impossibile* syllogism. Conversely any conclusion that can be derived by a *per impossibile* syllogism can also be derived, from the same premisses, by a direct syllogism (*An. Pr.* 45a26, 62b39; *An. Pr.* B11–14).[16] This distinction therefore requires that one attribute to the syllogism an argumentative structure which a conditional lacks.[17]

[13] Łukasiewicz, *Aristotle's Syllogistic*, pp. 20–30; Patzig, *Aristotle's Theory of the Syllogism*, pp. 3–4.

[14] Smiley, 'What is a syllogism?'; Corcoran, 'Aristotle's natural deduction system'.

[15] Clearly, Aristotle's *description* of a *per impossibile* syllogism differs from the traditional account of a *per impossibile* in which it is emphasized that one is deriving a contradiction from a supposition and a set of premisses. Cf. J. N. Keynes, *Studies and Exercises in Formal Logic*, section 257. For a discussion of this see Chapter 3 below.

[16] See Chapter 3.

[17] Further, the evidence Łukasiewicz and Patzig adduce is unconvincing. The evidence consists in the presence of the Greek word for 'if' (εἰ) before a statement of the premisses and the absence of the Greek word for 'therefore' (ἄρα) before the statement of the conclusion. However, that at a later time the word 'therefore' is conventionally used to mark that the conclusion of an inference is being drawn does not, of

A proof, for Aristotle, is a syllogism which enables one, simply by grasping it, to gain knowledge of the conclusion (*An. Pst.* 71b18ff). The premisses of a proof must possess certain important properties; for instance, they must be true, explanatory of, better known than and prior to the conclusion (*An. Pst.* 71b20). A syllogism in which the premisses had all the requisite properties would be a proof. It follows that a syllogism cannot merely consist in a relation of semantic consequence between premisses and conclusion. For if one simply states the axioms of a theory and a non-trivial semantic consequence *P*, there may be no way to tell whether *P* follows of necessity from the axioms. One cannot prove, for instance, that every triangle has interior angles equal to two right angles (Euclid I–32) merely by stating Euclid's postulates and then the theorem. A proof has a structure which reveals that the conclusion must be true if the premisses are. Therefore a syllogism must have a structure such that if the premisses had the appropriate properties one would be able, simply by studying the syllogism, to see that the conclusion is true. Must not a proof be a perfect (or perfected) syllogism? Curiously Aristotle does not mention this when discussing the properties a syllogism must have to be a proof (cf. *An. Pst.* A2–33). The reason, I think, is because every imperfect syllogism is perfectible. Any imperfect syllogism already has a structure such that it is possible to interpolate intermediate deductive steps designed to make it evident that the conclusion is a consequence of the premisses.

A syllogism should thus be thought of as a deduction, an entity which possesses a structural as well as a semantic relation between premisses and conclusion. Aristotle's project is to provide a formal analysis of the non-formal deductions with which he was familiar. Indeed, there is an ambiguity in Aristotle's use of the word 'syllogism' similar to that in the modern use of the word 'deduction'. There is, first, the use of 'syllogism' in the broad sense of the definition as a *logos* in which, certain things being posited, something other than what is

course, imply that at a period before the convention is in use the absence of 'therefore' should be taken as evidence that an inference is not being made. Further, there is no need to take 'if' (εἰ) as the hallmark of a conditional: it could equally well signify that the premisses are to be supposed or entertained. Even in contemporary English, the use of 'If . . . then . . .' is neither criterion that one has stated a conditional nor that one has not drawn an inference. If one were actually reasoning validly, rather than remarking on the validity of an inference, the use of 'If . . . then . . .' would not be unnatural. Whether or not the statement should be interpreted as a conditional or an inference would depend upon the context of utterance and not merely on that particular mode of expression. For a development of this and other criticisms of Łukasiewicz, see Smiley, 'What is a syllogism?'.