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A. G. Hamilton

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AN INTRODUCTION
WITH CONCURRENT EXAMPLES

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PREFACE

My earlier book, *A First Course in Linear Algebra with Concurrent Examples* (referred to below as the First Course), was an introduction to the use of vectors and matrices in the solution of sets of simultaneous linear equations and in the geometry of two and three dimensions. As its name suggests, that much is only a start. For many readers, such elementary material may satisfy the need for appropriate mathematical tools. But, for others, more advanced techniques may be required, or, indeed, further study of algebra for its own sake may be the objective.

This book is therefore in the literal sense an extension of the First Course. The first eleven chapters are identical to the earlier book. The remainder forms a sequel: a continuation into the next stage of the subject. This aims to provide a practical introduction to perhaps the most important applicable idea of linear algebra, namely eigenvalues and eigenvectors of matrices. This requires an introduction to some general ideas about vector spaces. But this is not a book about vector spaces in the abstract. The notions of subspace, basis and dimension are all dealt with in the concrete context of n -dimensional real Euclidean space. Much attention is paid to the diagonalisation of real symmetric matrices, and the final two chapters illustrate applications to geometry and to differential equations.

The organisation and presentation of the content of the First Course were unusual. This book has the same features, and for the same reasons. These reasons were described in the preface to the First Course in the following four paragraphs, which apply equally to this extended volume.

‘Learning is not easy (not for most people, anyway). It is, of course, aided by being taught, but it is by no means only a passive exercise. One who hopes to learn must work at it actively. My intention in writing this book is not to teach, but rather to provide a stimulus and a medium through which a reader can learn. There are various sorts of textbook with widely differing approaches. There is the encyclopaedic sort, which tends to be unreadable but contains all of the information relevant to its subject. And at the other extreme there is the work-book, which leads the reader in a progressive series of exercises. In the field of linear algebra

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there are already enough books of the former kind, so this book is aimed away from that end of the spectrum. But it is not a work-book, neither is it comprehensive. It is a book to be worked through, however. It is intended to be read, not referred to.

‘Of course, in a subject such as this, reading is not enough. Doing is also necessary. And doing is one of the main emphases of the book. It is about methods and their application. There are three aspects of this provided by this book: description, worked examples and exercises. All three are important, but I would stress that the most important of these is the exercises. You do not know it until you can do it.

‘The format of the book perhaps requires some explanation. The worked examples are integrated with the text, and the careful reader will follow the examples through at the same time as reading the descriptive material. To facilitate this, the text appears on the right-hand pages only, and the examples on the left-hand pages. Thus the text and corresponding examples are visible simultaneously, with neither interrupting the other. Each chapter concludes with a set of exercises covering specifically the material of that chapter. At the end of the book there is a set of sample examination questions covering the material of the whole book.

‘The prerequisites required for reading this book are few. It is an introduction to the subject, and so requires only experience with methods of arithmetic, simple algebra and basic geometry. It deliberately avoids mathematical sophistication, but it presents the basis of the subject in a way which can be built on subsequently, either with a view to applications or with the development of the abstract ideas as the principal consideration.’

Last, this book would not have been produced had it not been for the advice and encouragement of David Tranah of Cambridge University Press. My thanks go to him, and to his anonymous referees, for many helpful comments and suggestions.