A first course in linear algebra

A FIRST COURSE IN

linear algebra

WITH CONCURRENT EXAMPLES

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PREFACE

Learning is not easy (not for most people, anyway). It is, of course, aided by being taught, but it is by no means only a passive exercise. One who hopes to learn must work at it actively. My intention in writing this book is not to teach, but rather to provide a stimulus and a medium through which a reader can learn. There are various sorts of textbooks with widely differing approaches. There is the encyclopaedic sort, which tends to be unreadable but contains all of the information relevant to its subject. And at the other extreme there is the work-book, which leads the reader through a progressive series of exercises. In the field of linear algebra there are already enough books of the former kind, so this book is aimed away from that end of the spectrum. But it is not a work-book, neither is it comprehensive. It is a book to be worked through, however. It is intended to be read, not referred to.

Of course, in a subject such as this, reading is not enough. Doing is also necessary. And doing is one of the main emphases of the book. It is about methods and their application. There are three aspects of this provided by this book: description, worked examples and exercises. All three are important, but I would stress that the most important of these is the exercises. In mathematics you do not know something until you can do it.

The format of the book perhaps requires some explanation. The worked examples are integrated with the text, and the careful reader will follow the examples through at the same time as reading the descriptive material. To facilitate this, the text appears on the right-hand pages only, and the examples on the left-hand pages. Thus the text and corresponding examples are visible simultaneously, with neither interrupting the other. Each chapter concludes with a set of exercises covering specifically the material of that chapter. At the end of the book there is a set of sample examination questions covering the material of the whole book.

The prerequisites required for reading this book are few. It is an introduction to the subject, and so requires only experience with methods of arithmetic, simple algebra and basic geometry. It deliberately avoids mathematical sophistication, but it presents the basis of the subject in a way which can be built on subsequently, either with a view to applications or with the development of the abstract ideas as the principal consideration.

Examples

1.1 Simple elimination (two equations).

$$2x+3y=1$$
$$x-2y=4.$$

Eliminate x as follows. Multiply the second equation by 2:

$$2x+3y=1$$
$$2x-4y=8.$$

Now replace the second equation by the equation obtained by subtracting the first equation from the second:

$$2x+3y=1$$
$$-7y=7.$$

Solve the second equation for y, giving $y=-1$. Substitute this into the first equation:

$$2x-3=1,$$

which yields $x=2$. Solution: $x=2$, $y=-1$.

1.2 Simple elimination (three equations).

$$x-2y+z=5$$
$$3x+y-z=0$$
$$x+3y+2z=2.$$

Eliminate z from the first two equations by adding them:

$$4x-y=5.$$

Next eliminate z from the second and third equations by adding twice the second to the third:

$$7x+5y=2.$$

Now solve the two simultaneous equations:

$$4x-y=5$$
$$7x+5y=2$$

as in Example 1.1. One way is to add five times the first to the second, obtaining

$$27x=27,$$

so that $x=1$. Substitute this into one of the set of two equations above which involve only x and y, to obtain (say)

$$4-y=5,$$

so that $y=-1$. Last, substitute $x=1$ and $y=-1$ into one of the original equations, obtaining

$$1+2+z=5,$$

so that $z=2$. Solution: $x=1$, $y=-1$, $z=2$.