

## **Contents**

orewor	d to volume II, with an example for the end of volume I	хi
Errata f	or volume I	xxv
IX	Jensen's Formula Again	1
Α	Pólya's gap theorem	1
В	Scholium. A converse to Pólya's gap theorem	8
	1 Special case. $\Sigma$ measurable and of density $D > 0$	8
	Problem 29	9
	2 General case; Σ not measurable. Beginning of Fuchs' construction	13
	3 Bringing in the gamma function	20
	Problem 30	22
	4 Formation of the group products $R_{j}(z)$	24
	5 Behaviour of $(1/x)\log (x-\lambda)/(x+\lambda) $	29
	6 Behaviour of $(1/x)\log  R_j(x) $ outside the interval $[X_j, Y_j]$	31
	7 Behaviour of $(1/x)\log  R_j(x) $ inside $[X_j, Y_j]$	34
	8 Formation of Fuchs' function $\Phi(z)$ . Discussion	43
	9 Converse of Pólya's gap theorem in general case	52
С	A Jensen formula involving confocal ellipses instead of circles	57
D	A condition for completeness of a collection of imaginary	
	exponentials on a finite interval	62
	Problem 31	64
	1 Application of the formula from §C	65
	2 Beurling and Malliavin's effective density $\tilde{D}_{\Lambda}$	70
E	Extension of the results in §D to the zero distribution of entire	
	functions $f(z)$ of exponential type with	
	$\int_{-\infty}^{\infty} (\log^+  f(x) /(1+x^2)) dx  \text{convergent}$	87
	1 Introduction to extremal length and to its use in estimating	
	harmonic measure	88
	Problem 32	101
	Problem 33	108
	Problem 34	109



## viii Contents

	2 Real zeros of functions $f(z)$ of exponential type with	
	$\int_{-\infty}^{\infty} (\log^+  f(x) /(1+x^2))  \mathrm{d}x < \infty$	110
F	Scholium. Extension of results in §E.1. Pfluger's theorem and	
	Tsuji's inequality	126
	1 Logarithmic capacity and the conductor potential	127
	Problem 35	131
	2 A conformal mapping. Pfluger's theorem	132
	3 Application to the estimation of harmonic measure. Tsuji's	
	inequality	140
	Problem 36	146
	Problem 37	157
X	Why we want to have multiplier theorems	158
Α	Meaning of term 'multiplier theorem' in this book	158
	Problem 38	159
	1 The weight is even and increasing on the positive real axis	159
_	2 Statement of the Beurling-Malliavin multiplier theorem	164
В	Completeness of sets of exponentials on finite intervals	165
	1 The Hadamard product over $\Sigma$	169
	2 The little multiplier theorem	173
	3 Determination of the completeness radius for real and complex	100
	sequences A Problem 39	189 195
С	The multiplier theorem for weights with uniformly continuous	193
~	logarithms	195
	1 The multiplier theorem	195
	2 A theorem of Beurling	202
	Problem 40	208
D	Poisson integrals of certain functions having given weighted	
	quadratic norms	209
Ε	Hilbert transforms of certain functions having given weighted	
	quadratic norms	225
	1 $H_p$ spaces for people who don't want to really learn about them	226
	Problem 41	234
	Problem 42	248
	2 Statement of the problem, and simple reductions of it	249
	3 Application of $H_p$ space theory; use of duality	260
	4 Solution of our problem in terms of multipliers	272
_	Problem 43	279
F	Relation of material in preceding § to the geometry of unit sphere	
	$\ln L_{\infty}/H_{\infty}$	282
	Problem 44	292
	Problem 45	293
	Problem 46	295
	Problem 47	296



	Contents	ix
ΧI	Multiplier theorems	298
Α	Some rudimentary potential theory	298
	1 Superharmonic functions; their basic properties	298
	2 The Riesz representation of superharmonic functions	311
	Problem 48	327
	Problem 49	328
	3 A maximum principle for pure logarithmic potentials.	
	Continuity of such a potential when its restriction to generating	
	measure's support has that property	329
	Problem 50	334
	Problem 51	339
В	Relation of the existence of multipliers to the finitness of a	
_	superharmonic majorant	341
	1 Discussion of a certain regularity condition on weights	341
	Problem 52	361
	Problem 53	362
	2 The smallest superharmonic majorant	363
	Problem 54	369
	Problem 55	370
	Problem 56	371
	3 How $\mathfrak{M}F$ gives us a multiplier if it is finite	374
	Problem 57	383
С	Theorems of Beurling and Malliavin	389
_	1 Use of the domains from §C of Chapter VIII	391
	2 Weight is the modulus of an entire function of exponential type	395
	Problem 58	405
	3 A quantitative version of the preceding result	407
	Problem 59	412
	Problem 60	413
	4 Still more about the energy. Description of the Hilbert space §	
	used in Chapter VIII, §C.5	418
	Problem 61	443
	Problem 62	444
	5 Even weights W with $\ \log W(x)/x\ _{\mathcal{E}} < \infty$	446
	Problem 63	451
	Problem 64	452
D	Search for the presumed essential condition	452
	1 Example. Uniform Lip 1 condition on $\log \log W(x)$ not	
	sufficient	454
	2 Discussion	467
	Problem 65	469
	3 Comparison of energies	472
	Problem 66	483
	Problem 67	484



## x Contents

	Problem 68	487
	4 Example. The finite energy condition not necessary	487
	5 Further discussion and a conjecture	502
E	A necessary and sufficient condition for weights meeting the local	
	regularity requirement	511
	1 Five lemmas	512
	2 Proof of the conjecture from §D.5	524
	Problem 69	558
	Problem 70	561
	Problem 71	565
	Bibliography for volume II	566
	Index	572