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*PRINCIPLES OF EARTHQUAKE
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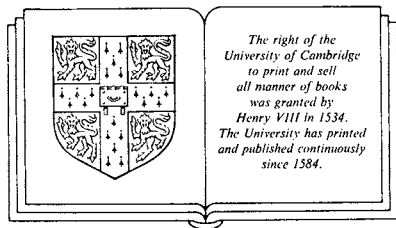
Principles of earthquake source mechanics

B. V. KOSTROV

*Institute of Physics of the Earth,
Academy of Sciences, Moscow*

SHAMITA DAS

*Lamont-Doherty Geological Observatory
of Columbia University*



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SYMBOLS

Symbols used frequently in the book are listed below; the list is not an exhaustive one. Occasionally the same symbol represents different quantities in different contexts, but we believe that no confusion should arise because the meaning of the symbol will be clear from the context. The equation number given with a definition identifies either the equation in which the symbol is defined or the equation in which it is first used.

a_0	Critical slip displacement in crack tip process zone
$\dot{\mathbf{a}}(\mathbf{x}, t)$	Slip velocity vector at (\mathbf{x}, t) on the fault (crack)
\bar{a}_i	Average or smoothed slip over fault
$\bar{a}_{i(\kappa)}$	Average of the i th component of slip over the fault due to the κ th earthquake, $\kappa = 1, 2, \dots, N$, $i = 1, 2$ [equation (4.5.1)]
$a_{i(\kappa)}(\mathbf{x})$	i th component of final slip (after earthquake) at \mathbf{x} for the κ th earthquake, $\kappa = 1, 2, \dots, N$, $i = 1, 2$ [equation (4.5.1)]
$a_i(\mathbf{x}, t)$	Displacement jump across fault at (\mathbf{x}, t) , $i = 1, 2$
$a_{(n)}$	Normal component of displacement jump across fault (opening displacement)
$A^P(t, \mathbf{m})$	P-Wave pulse shape at the observation point in the direction \mathbf{m} from the source [equations (3.3.20) and (3.3.21)]
$A_k^S(t, \mathbf{m})$	k th component of the S-wave pulse shape at the observation point in the direction \mathbf{m} from the source [equation (3.3.21)]

viii *Symbols*

$A^{\text{SV}}(t, \mathbf{m}),$ $A^{\text{SH}}(t, \mathbf{m})$	SV- and SH-wave pulse shapes at the observation point in the direction \mathbf{m} from the source [equations (3.3.22) and (3.3.23)]
$A_0^{\text{P}}(\mathbf{m})$	P-Wave radiation pattern for a point source [equation (3.4.5)]
$A_{0k}^{\text{S}}(\mathbf{m})$	k th component of the S-wave radiation pattern for a point source [equation (3.4.5)]
$A_0^{Q,n}$	Observed amplitude at station n for wave type Q at low frequency [equations (4.2.4) and (4.2.5)]
$A_{lm}^{Q,n}(\omega)$	Displacement spectrum for various components for wave type Q for station n
$A_{0lm}^{Q,n}$	Value to which $A_{lm}^{Q,n}(\omega)$ tends as $\omega \rightarrow 0$ [equations (4.2.4) and (4.2.5)]
b_i	Unit vector
$b_{i(\kappa)}$	Unit vector corresponding to the i th slip component for the κ th source, $\kappa = 1, 2, \dots, N$, $i = 1, 2$ [equation (4.5.1)]
c	Dimensionless constant, depending on fault geometry and material properties [equation (4.3.3)]
c_{ijkl}	Stiffness tensor [equation (1.3.17)]
d	Maximum dimension of fracture
$D_{ki}^{\text{P},\text{S}}(\mathbf{m})$	P- and S-wave radiation patterns for the single-asperity model [equation (4.6.4)]
e_{klm}	Unit antisymmetric tensor [equation (3.5.11)]
E_q	Seismic energy on an earthquake [equation (4.3.7)]
f	Some function of one or more variables
\tilde{f}	Function defined by equation (3.7.1)
f_{MN}^a	Function defined by equation (3.7.6) and the equation preceding it
F_0	Positive constant independent of grid size representing the discretized value of $G_{\alpha\beta}(\mathbf{x}, t)$ at (\mathbf{x}, t) [equation (5.2.7) and Appendix 2]
$F_{\text{P}}(t, \mathbf{m}),$ $F_{\text{S}}(t, \mathbf{m})$	Functions of time whose dependence on the ray direction \mathbf{m} determines the ratio of the actual source radiation to that of the concentrated dipole for P and S waves, respectively [equations (3.4.6) and (3.4.7)]
$F_{\alpha\beta}$	Discretized form of the kernel $G_{\alpha\beta}(\mathbf{x}, t)$ [equation (5.2.4)]
$\mathcal{F}^{\text{P}}(\mathbf{m})$	Function defined by equation (6.1.4)

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G	Crack-driving force
G_0	Positive constant used in stabilizing the solution of the boundary–integral equation [(5.2.9) and Appendix 2]
G_i	Crack-driving force for Mode I ($i = 1$), Mode II ($i = 2$), and Mode III ($i = 3$) cracks
G_c	Critical G for fracture initiation
G_d	Critical G for fracture arrest
$G_{ik}(\mathbf{x}, t)$	k th component of the Green function (impulse response) of an elastic medium due to a point force directed along x_i , $i, k = 1, 2, 3$ (expressions for particular cases in Appendix 1)
h	P-Indicator function [equation (3.6.5)]
h'_{ϕ_0}	Derivative of h with respect to ϕ_0 [equation (3.6.13)]
$H(\)$	Heaviside function
k	Coefficient of friction or bulk modulus
k_i	Stress intensity factors, $i = 1$ for Mode I (tension) crack, $i = 2$ for Mode II (inplane shear) crack, $i = 3$ for Mode III (antiplane shear) crack [equation (2.1.2)]
k_p, k_s	Wave number for P and S waves, respectively
k_{ic}	Critical value of the stress intensity factor k_i at which fracture occurs, $i = 1, 2, 3$
$k(\phi)$	Support function [equation (3.6.5)]
K_{ik}	Kernel of integral equation, defined by equation (3.2.19)
$K_{\alpha 3k}$	Kernel of integral equation, defined by equation (5.1.2), $\alpha = 1, 2$
$K(\mathbf{x}, t), K(t)$	Smoothing kernels for slip distribution on fault
l	Some dimension of the fracture area
L_0	Length of initial defect [equation (2.4.11)]
L_c	Critical half-length of Griffith inplane shear crack
\mathbf{m}	(m_1, m_2, m_3) , unit vector at source directed along the ray to receiver
\mathbf{m}_1	(m_{11}, m_{12}, m_{13}) , unit vector at receiver directed along ray to the source
\mathbf{m}'	$(-m_2, m_1, m_3)$
m_i^{\parallel}	Component of \mathbf{m} lying in the fault plane [equation (3.4.8)]
m_i^{\perp}	Component of \mathbf{m} orthogonal to the fault plane [equation (3.4.8)]
\mathbf{m}^{\parallel}	Unit vector defined by equation (3.5.4)

<i>x</i>	<i>Symbols</i>
$\mathbf{m}_P, \mathbf{m}_S$	\mathbf{m} for P and S waves, respectively
m_{lm}	Components of the seismic moment tensor for a general source where the force is directed along the x_l axis and the arm of the couple along the x_m axis, $l, m = 1, 2, 3$, [equation (3.2.13)]; for an isotropic medium, equation (3.2.17)]
M_0	Seismic moment [equation (4.1.20)]
M_k	Mechanical moment [equation (3.2.15)]
$M_{lm}(\omega)$	Components of the seismic moment tensor for a concentrated dipole at frequency ω
M_{0lm}	Seismic moment tensor components as $\omega \rightarrow 0$ [equation (4.1.13)]
$M_{0(\kappa)}$	Seismic moment of the κ th earthquake, $\kappa = 1, 2, \dots, N$ [equation (4.5.3)]
$M_{0lm(\kappa)}$	M_{0lm} for the κ th earthquake, $\kappa = 1, 2, \dots, N$ [equation (4.5.14)]
\mathbf{n}	Unit vector, generally directed along the normal to the fault plane
OV	Dynamic overshoot in slip at the center of crack
p_i	Ray parameter; $= m_i/v_P$ and $= m_i/v_S$ for P and S waves, respectively
P_α	Geometrical constant related to an elliptical asperity
P_{MN}	Polynomial of degree $(M + N)$ [equation (3.7.6)]
$\mathcal{P}^P(\mathbf{m})$	P-Radiation pattern due to a concentrated dipole
q	Specific heat production [equation (2.2.3)]
q_i	Heat flux vector [equation (2.2.3)]
Δq	Heat flux from unit area of fault [equation (2.2.15)]
ΔQ_r	Radiation loss [equation (4.4.27)]
r	Final crack radius, or distance from crack tip [equation (2.1.2)], or distance from source to receiver, or radius of centered asperity in model of fracture of single-asperity on circular fault [equation (6.3.3)]
r_0	Initial crack radius
R	Distance from fracture initiation point to receiver or radius of outer circular crack in model of single-asperity failure [equation (6.3.1)]
R_P, R_{SV}, R_{SH}	Geometrical spreading factor for P, SV, and SH waves, respectively [equations (3.3.20), (3.3.22), and (3.3.23)]
$R(\omega, \xi)$	Form of the Rayleigh function in (ω, ξ) space [equation (3.5.13)]

Symbols

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S	Measure of fault strength = $(\sigma^u - \sigma^0)/\Delta\sigma$, or area of fault (crack), or boundary of elastic volume V
S_0	Surface area of source or value of tip element $S_{\alpha\beta}(x_1, x_2, t)$ at $x_1 = 0, x_2 = 0, t = 0$ [equation (5.2.13)]
$S_{(k)}$	Union of all grids with nonzero slip (discrete slipping area) [equation (5.2.13)]
$S_{(k)}^P$	Union of all grids influenced by disturbances at time $k \Delta t$ [equation (5.2.9)]
$S_{\alpha\beta}$	Numerical counterpart of $T_{\alpha\beta}$, $\alpha, \beta = 1, 2$ [equations (5.2.11) and (5.2.12)]
t, t_1	Times (e.g., times at source and receiver)
t_m	Source duration or maximum source duration
Δt	Temporal grid size in numerical solution of boundary-integral equation (5.2.1)
T	$v_p t / \Delta x$ or temperature [equation (2.2.17)]
T_P, T_S	P and S travel times [equation (3.3.13)]
$T_{\alpha\beta}(\mathbf{x}, t)$	Integral equation kernel defined by equation (5.1.4)
u_0	Applied shift at $\pm\infty$ in problems of failure of asperity on an infinite fault
$u_i(\mathbf{x}, t)$	Displacement components at (\mathbf{x}, t) , $i = 1, 2, 3$
u^\pm	Displacement components on + and - sides of fault [equation (1.3.25)]
$\dot{u}_i(\mathbf{x}, t)$	Velocity components
$\ddot{u}_i(\mathbf{x}, t)$	Acceleration components
$u_k^P(\mathbf{x}, t),$ $u_k^S(\mathbf{x}, t)$	k th component of P and S far-field displacements, $k = 1, 2, 3$ [equations (3.3.4) and (3.3.5)]
$u_k^P(\ddot{\mathbf{x}}, t),$ $u_k^S(\ddot{\mathbf{x}}, t)$	k th component of P and S accelerations, $k = 1, 2, 3$
$u_k^{SV}(\mathbf{x}, t),$ $u_k^{SH}(\mathbf{x}, t)$	k th component of SV and SH far-field displacements, $k = 1, 2, 3$
u_{SV1}, u_{SH1}	SV and SH displacements at the observation point [equations (3.3.22) and (3.3.23)]
u_{SV}, u_{SH}	SV and SH displacements at the source point [equations (3.3.25) and (3.3.26)]
U_{ik}	Green's tensor for elastic problems [solution of equation (3.1.6)]; for a homogeneous, infinite medium, equation (3.3.1)]
U_{ik}^P, U_{ik}^S	P- and S-wave contributions to U_{ik}
v	Fracture speed

xii	<i>Symbols</i>
v_1, v_2	Fracture speeds in x_1 and x_2 directions, respectively [equation (3.4.24)]
v_H	Speed of healing wave on crack
v_P	Compressional wave speed of elastic medium
v_R	Rayleigh wave speed of elastic half-space
v_S	Shear wave speed of elastic medium
v_{\max}	Maximum fracture speed
V	Volume
V^\pm	Volume of integration on + and - sides of the fault
ΔV	Change in volume
w_0	Initial potential energy density
w^1	Final elastic energy density
w_e	Strain energy density [equation (1.3.22)]
\dot{W}	Rate of change of elastic energy within some volume [equation (4.4.13)]
\mathbf{x}	x_i , a Cartesian coordinate system, $i = 1, 2, 3$, or source coordinates
\mathbf{x}_1	Receiver coordinates
Δx	Spatial grid size in numerical solution of boundary-integral equation (5.2.1)
\mathbf{z}	Unit vector at receiver directed along vertical
\mathbf{z}_1	Unit vector at source directed along vertical
α, β	Subscripts with a value of 1 or 2
γ	Specific surface energy [equation (2.1.1)]
γ_{eff}	Effective γ
δ	Crack opening displacement at which cohesive forces at crack tip vanish [equation (2.1.7)]
δ_{ij}	Kronecker delta ($= 1$ for $i = j$; $= 0$ for $i \neq j$)
$\delta(\mathbf{x}), \delta(t)$	Dirac deltas
$\Delta(v)$	Rayleigh function [equation (2.3.22)]
ϵ	An arbitrarily small number
$\epsilon_{kl}, \epsilon'_{kl}$	Strain components, $k, l = 1, 2, 3$
$\dot{\epsilon}_{(\kappa)kl}$	Average seismic strain rate due to the κ th earthquake, $\kappa = 1, 2, \dots, N$
$\overline{\Delta\epsilon_{kl}}$	Average seismic strain rate tensor [equations (4.5.12) and (4.5.13)]
θ	Angle between direction to source and normal to fault [equation (6.2.1)], or $(90^\circ - \theta)$ [equation (3.4.23)], or angle that applied shift at infinity makes with x_1 axis in elliptical-asperity failure problem

Symbols

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λ	Lamé constant
λ_{\min}	Minimum wavelength recorded [equation (3.3.40)]
λ_P, λ_S	P and S wavelengths [equation (4.1.3)]
μ	Modulus of rigidity
ν	Poisson's ratio
ξ	ξ_i is a Cartesian coordinate system, $i = 1, 2, 3$, or $\xi_1 = \omega p_1$ and $\xi_2 = \omega p_2$ [equation (3.8.6)]
ρ	Density
σ^0	Initial stress (before earthquake)
σ^1	Final stress (after earthquake)
σ^u	Critical stress required for failure of nonideally brittle body
σ^{inf}	Stress applied at infinity
σ^{stat}	Static frictional stress
σ^{kin}	Kinetic frictional stress
σ_B	Strength of laboratory rock sample [equation (2.4.15)]
σ_r	Average radiational friction
$\Delta\sigma$	Stress drop on crack (fault)
$\sigma_{ik}, \sigma'_{ik}$	Stress components
σ_{ik}^0	Component of initial stress
Σ_{ijk}	Stress tensor corresponding to Green's tensor U_{ik} [equation (3.1.9)]
$\Sigma(t)$	Fault area at time t
τ_0, τ_1	Pulse durations
τ_{ik}	Components of stress perturbations [equation (1.3.19)]
τ_{ik}^{\pm}	τ_{ik} on + and - sides of fault [equation (1.3.25)]
ϕ	Polar angle [equation (3.6.5)]
ϕ_0	Polar angle [equation (3.6.12)]
Ψ_{MN}^a	Function defined by equation (3.7.7)
ω	Angular frequency
'	Derivative with respect to following variables
.	Derivative with respect to time
[]	Jump of variable contained within the square brackets across any discontinuity

PREFACE

The practical goal of earthquake seismology is to prevent or reduce losses due to earthquakes by estimating the earthquake hazard at a given site or by forecasting the occurrence of the next strong event. The prevailing approach to these problems is to extrapolate data from the record of past events and apply that information to the future. Acceleration spectra of strong earthquakes are used to design structures, and unusual phenomena observed before earthquakes are considered possible indicators of future ones. This purely phenomenological approach is unreliable, due mainly to the lack of representative data. Recurrence times of strong earthquakes are fortunately very long, namely, several tens or hundreds of years, whereas the subject of seismology is only approaching its centennial. Obviously, an understanding of the earthquake generation process can partly fill the gaps in the available data, thereby making the practical conclusions more reliable. This is the purpose of earthquake source studies.

As is usual in geophysics, one can neither experiment with nor directly observe the earthquake focal region; one can only investigate its manifestations at the earth's surface. To convert these manifestations into information on the earthquake source process, one must have some general theoretical model that can then be fit to observational data. The history of the development of the source model is detailed in the Introduction. Making use of recent achievements in rock mechanics as well as progress in geophysical observations, the model has developed during the past several decades from the double-couple point source to the very sophisticated picture of the fracturing process at inhomogeneous

faults. The complexity of the model reflects the complexity of the earthquake source process itself. It must be noted that oversimplification of the model may have unfortunate consequences for its practical application, as when catastrophic events are falsely predicted, mainly on the basis of theoretical speculations. At the same time, our current understanding of the complexity of the earthquake source makes it difficult to draw definite conclusions in view of the rather scarce observational information. The rapid development of geophysical data acquisition systems promises to improve the situation in the not too distant future. In any case, further refinement of the model is largely speculative. So although earthquake source theory is far from being complete or even consistent, it seems appropriate to summarize the more mature aspects of the state of the art. This is what this book is intended to do. More precisely, we confine the subject to the most basic principles of the theory and consider in some detail only the subject of earthquake dynamics, that is, the process of rupturing and the resulting seismic radiation during an earthquake, leaving aside the theory of earthquake preparation and interaction, which is at a less developed stage. We also include some mathematical tools for earthquake modeling. Our approach is purely deterministic, and no attention has been paid to the implications of the randomness of either the earth's structure or the source excitation process. Like Perry Mason, we consider only "possibilities," not "probabilities." The book deals only in concepts. Data-processing techniques used in seismology are mentioned purely for the purpose of illustration and are presented in only extremely simplified form.

The book consists of three parts, the first two of which are adapted from the Russian monograph *Mechanics of the Tectonic Earthquake Focus* by B. V. Kostrov, which was published in 1975 by Nauka, Moscow. These parts have been heavily rewritten and updated, and several new sections have been added. The third part is new. The book is meant primarily for use by graduate students and by teachers of earthquake source theory. We hope that the material will also aid the development of data-acquisition systems and data-processing techniques.

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Preface

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of the manuscript, and provided original copies of their figures. In particular, Kei Aki, Jim Rice, Jo Andrews, Jo Walsh, and Teng-fong Wong gave us their opinions, and Jim Dieterich and Barry Raleigh helped make it possible for us to write the book. We express our sincere gratitude to them.