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More information

Symbol Index

q Flow rate of heat energy

k Thermal conductivity

$u_x \equiv \dfrac{\partial u}{\partial x}$ Partial derivative of u with respect to x

$u_t \equiv \dfrac{\partial u}{\partial t}$ Partial derivative of u with respect to t

$u_{xx} \equiv \dfrac{\partial^2 u}{\partial x^2}$ Second partial derivative of u with respect to x

c Heat capacity

ρ Density of the conductor

Δx Change in x

Δt Change in t

κ Thermal diffusivity $(\rho c)^{-1}k$

$\exp\{x\} \equiv e^x$, $e = 2.71828\ldots$

$\sin x \equiv \dfrac{e^{ix} - e^{-ix}}{2i}$, $i = \sqrt{-1}$

$\cos x \equiv \dfrac{e^{ix} + e^{-ix}}{2}$

$\sinh x \equiv \dfrac{e^{x} - e^{-x}}{2}$

$\cosh x \equiv \dfrac{e^x + e^{-x}}{2}$

$p_n(x,t) \equiv n! \sum_{k=0}^{[n/2]} \dfrac{t^k}{k!} \dfrac{x^{n-2k}}{(n-2k)!}$ Heat polynomials

$\Gamma(a) = \int_0^\infty x^{a-1} e^{-x}\, dx,\ 0 < a < \infty$ The gamma function

$n!$ $1 \cdot 2 \cdot 3 \cdots n = \Gamma(n+1)$

$\mathbb{R}$ Real numbers

D Open domain in $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$

D_T $= D \cap \{(x,t) \in \mathbb{R}^2 \mid t \le T\} \cup$ certain boundary points (x, T) to form the parabolic domain

B_T The parabolic boundary of D_T

$\in$ Belongs to

ε Positive real number

$\subset$ Inclusion of a set in a set. For example, for $A \subset B$ read: A is a subset of B.

$\cup$ Union of sets

$\cap$ Intersection of sets

$\{x \in A \mid P(x)\}$ Set-builder notation: $x \in A$ such that property $P(x)$ is satisfied.

$f^{(j)}(t)$ jth derivative of f with respect to its argument

$K(x,t) \equiv (4\pi t)^{-1/2} \exp\left\{-\dfrac{x^2}{4t}\right\},\ t > 0$ The fundamental solution of the heat equation

$G(x,\xi,t) \equiv K(x-\xi,t) - K(x+\xi,t)$ The Green's function for the semi-infinite conductor

$N(x,\xi,t) \equiv K(x-\xi,t) + K(x+\xi,t)$ The Neumann function for the semi-infinite conductor

$\theta(x,t) = \sum_{n=-\infty}^{\infty} K(x+2n,t)$ The theta function

$\|\psi\|_t = \sup_{0<\tau<t} |\psi(\tau)|$ or $\max_i \{\|\psi_i\|_t\}$ for a vector function ψ

$\|f\|_{[a,b]} = \sup_{a \le x \le b} |f(x)|$

$\|E\|_j = \max_{0 \le i \le m} |E_{i,j}|$ Vector norm for the vector $E \equiv E_j = (E_{i,j}, \ldots, E_{m,j})$, $E_{i,j} \in \mathbb{R}$.

$\|\psi\|_2 = \left\{\int_D |\psi|^2\right\}^{1/2}$ The L^2 norm

C, C_i, $i =$ positive integer Positive constants

$C(I)$ Where I is an interval or domain, denotes the continuous functions on I

$C^1(I)$ Denotes the space of continuously differentiable functions on I

$C^\beta(I)$, $0 < \beta \le 1$ Denotes the space of Hölder continuous functions on I with Hölder exponent β. The space of Lipschitz continuous functions, $\beta = 1$, presents some confusion which is removed by the context of the usage below.

$|\psi|_\beta = \sup\limits_{\substack{t,t+\delta\in I\\ \delta>0}} \delta^{-\beta}|\psi(t+\delta)-\psi(t)|$ The Hölder seminorm or constant for ψ

$C^0_{(\nu)}(I)$, $0<\nu\le 1$ The subspace of $C(I)$, $I=(0,T]$, such that $\|\psi\|^{(\nu)}_T = \sup\limits_{t\in I} t^{1-\nu}|\psi(t)| < \infty$

$C^\varepsilon_{(\nu)}(I)$, $\varepsilon>0$ The subspace of $C^0_{(\nu)}(I)$, $I=(0,T]$, such that

$$|\psi|^{(\nu)}_\varepsilon = \sup_{\substack{t,t+\delta\in I\\ \delta>0}} t^{1-\nu+\varepsilon}\delta^{-\varepsilon}|\psi(t+\delta)-\psi(t)| < \infty$$

P Used in Chapter 16 to denote a closed polygonal region

Q Used in Chapter 1 to denote heat content and in Chapter 16 to denote a point (x,t)

$\mathscr{L}(u)\equiv u_{xx}-u_t$ Used to assist in the repetitive usage of the heat operator

$M_P\psi$ Denotes the function used in Chapter 16 formed from the function ψ defined in D as follows:

$$M_P\psi(Q) = \begin{cases} \psi, & Q\in D-P^0\ (P^0 \text{ is the interior of } P), \\ \mathscr{L}(\psi)=0, & Q\in P^0. \end{cases}$$

$s(h,t) = \sup\limits_{0<\tau<t} h(\tau)$ Used in Chapter 13

$i(h,t) = \inf\limits_{0<\tau<t} h(\tau)$ Also used in Chapter 13

$\dot{s}(t)$ Derivative of s with respect to t

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