

# Contents

<b>Editor's Statement</b> .....	<b>xiii</b>
<b>Foreword</b> .....	<b>xv</b>
<b>Preface</b> .....	<b>xix</b>
<b>Chapter 0 Elements of Functional Analysis</b> .....	<b>1</b>
1 Banach and Hilbert Spaces. Bounded Operators. Integration of Vector-Valued Functions .....	1
2 Linear Functionals: The Dual Space. Vector-Valued Analytic Functions .....	4
3 Unbounded Operators; the Resolvent. Closed Operators .....	6
4 Adjoints .....	11
5 Generalized Sequences. Weak Convergence .....	15
6 Normal, Symmetric, Self-Adjoint, and Unitary Operators in Hilbert Space .....	16
7 Some Special Spaces and Their Duals .....	20
8 Convolution and Mollifiers. Sobolev Spaces .....	23
<b>Chapter 1 The Cauchy Problem for Some Equations of Mathematical Physics: The Abstract Cauchy Problem</b> .....	<b>26</b>
1.1 The Heat Equation in a Square .....	27
1.2 The Abstract Cauchy Problem .....	29

1.3	The Diffusion Equation in a Square . . . . .	32
1.4	The Schrödinger Equation . . . . .	39
1.5	The Cauchy Problem in $(-\infty, \infty)$ . . . . .	43
1.6	The Maxwell Equations . . . . .	43
1.7	Miscellaneous Notes . . . . .	54
<b>Chapter 2</b>	<b>Properly Posed Cauchy Problems: General Theory . . . . .</b>	<b>62</b>
2.1	The Cauchy Problem in $t \geq 0$ . . . . .	63
2.2	The Cauchy Problem in $-\infty < t < \infty$ . The Adjoint Equation . . . . .	71
2.3	Semigroup Theory . . . . .	80
2.4	The Inhomogeneous Equation . . . . .	86
2.5	Miscellaneous Comments . . . . .	91
<b>Chapter 3</b>	<b>Dissipative Operators and Applications . . . . .</b>	<b>117</b>
3.1	Dissipative Operators . . . . .	117
3.2	Ordinary Differential Operators in the Whole Line . . . . .	125
3.3	Ordinary Differential Operators in a Closed Interval. Semi-Infinite Intervals . . . . .	135
3.4	Ordinary Differential Operators in a Compact Interval . . . . .	142
3.5	Symmetric Hyperbolic Systems in the Whole Space . . . . .	146
3.6	Isometric Propagators and Conservative Operators. Dissipative Operators in Hilbert Space . . . . .	154
3.7	Differential Equations in Banach Lattices. Positive Solutions. Dispersive Operators . . . . .	159
3.8	Miscellaneous Comments . . . . .	166
<b>Chapter 4</b>	<b>Abstract Parabolic Equations: Applications to Second Order Parabolic Equations . . . . .</b>	<b>172</b>
4.1	Abstract Parabolic Equations . . . . .	173
4.2	Abstract Parabolic Equations; Analytic Propagators . . . . .	179
4.3	Applications to Ordinary Differential Operators . . . . .	187
4.4	Second Order Partial Differential Operators. Dissipativity . . . . .	199
4.5	Second Order Partial Differential Operators. Assignment of Boundary Conditions . . . . .	204
4.6	Second Order Partial Differential Operators. Construction of $m$ -Dissipative Extensions in $L^2$ . . . . .	214
4.7	Regularity Theorems . . . . .	222
4.8	Construction of $m$ -Dissipative Extensions in $L^p(\Omega)$ and $C(\bar{\Omega})$ . . . . .	235
4.9	Analyticity of Solution Operators . . . . .	255

Contents	xi
4.10 Positivity and Compactness of Solution Operators . . . . .	263
4.11 Miscellaneous Comments . . . . .	265
<b>Chapter 5 Perturbation and Approximation of Abstract Differential Equations . . . . .</b>	<b>267</b>
5.1 A Perturbation Result . . . . .	268
5.2 The Neutron Transport Equation . . . . .	272
5.3 Perturbation Results for Operators in $\mathcal{C}(1, 0)$ and in $\mathcal{A}$ . . . . .	286
5.4 Perturbation Results for the Schrödinger and Dirac Equations . . . . .	292
5.5 Second Order Differential Operators . . . . .	304
5.6 Symmetric Hyperbolic Systems in Sobolev Spaces . . . . .	306
5.7 Approximation of Abstract Differential Equations . . . . .	316
5.8 Approximation of Abstract Differential Equations by Finite Difference Equations . . . . .	329
5.9 Miscellaneous Comments . . . . .	336
<b>Chapter 6 Some Improperly Posed Cauchy Problems . . . . .</b>	<b>346</b>
6.1 Improperly Posed Problems . . . . .	346
6.2 The Reversed Cauchy Problem for Abstract Parabolic Equations . . . . .	354
6.3 Fractional Powers of Certain Unbounded Operators . . . . .	357
6.4 Fractional Powers of Certain Unbounded Operators (Continuation) . . . . .	365
6.5 An Application: The Incomplete Cauchy Problem . . . . .	372
6.6 Miscellaneous Comments . . . . .	374
<b>Chapter 7 The Abstract Cauchy Problem for Time-Dependent Equations . . . . .</b>	<b>381</b>
7.1 The Abstract Cauchy Problem for Time-Dependent Equations . . . . .	381
7.2 Abstract Parabolic Equations . . . . .	390
7.3 Abstract Parabolic Equations: Weak Solutions . . . . .	405
7.4 Abstract Parabolic Equations: The Analytic Case . . . . .	409
7.5 The Inhomogeneous Equation . . . . .	415
7.6 Parabolic Equations with Time-Dependent Coefficients . . . . .	419
7.7 The General Case . . . . .	424
7.8 Time-Dependent Symmetric Hyperbolic Systems in the Whole Space . . . . .	437
7.9 The Case where $D(A(t))$ is Independent of $t$ . The Inhomogeneous Equation . . . . .	451
7.10 Miscellaneous Comments . . . . .	455

<b>Chapter 8</b>	<b>The Cauchy Problem in the Sense of Vector-Valued Distributions</b>	<b>461</b>
8.1	Vector-Valued Distributions. Supports, Convergence, Structure Results	461
8.2	Vector-Valued Distributions. Convolution, Tempered Distributions, Laplace Transforms	468
8.3	Convolution and Translation Invariant Operators. Systems: The State Equation	477
8.4	The Cauchy Problem in the Sense of Distributions	482
8.5	The Abstract Parabolic Case	493
8.6	Applications: Extensions of the Notion of Properly Posed Cauchy Problem	497
8.7	Miscellaneous Comments	504
<b>References</b>		<b>510</b>
<b>Index</b>		<b>627</b>