

Cambridge University Press

978-0-521-30230-2 - Product Integration with Applications to Differential Equations

John D. Dollard and Charles N. Friedman

Table of Contents

[More information](#)

# Contents

Editor's Statement . . . . .	xi
Section Editor's Foreword . . . . .	xiii
Preface . . . . .	xix
<b>Chapter 1 Product Integration of Matrix-Valued Functions . . . . .</b>	<b>1</b>
0 Introduction . . . . .	1
1.1 Product Integration . . . . .	3
1.2 Product Integral Analysis of Linear Ordinary Differential Equations . . . . .	12
1.3 Further Properties of Product Integrals . . . . .	19
1.4 Estimates of Size, and the Product Integral as a Time-Ordered Exponential . . . . .	28
1.5 Dependence on a Parameter . . . . .	32
1.6 Improper Product Integration . . . . .	37
1.7 Alternative Definitions of the Product Integral . . . . .	50
1.8 Lebesgue-Integrable Functions . . . . .	54
Notes to Chapter 1 . . . . .	56
<b>Chapter 2 Contour Product Integration . . . . .</b>	<b>59</b>
0 Introduction . . . . .	59
2.1 The Definition of Contour Product Integrals . . . . .	59
2.2 The Product Integral of an Analytic Function and the Analogues of Cauchy's Integral Theorem . . . . .	62
2.3 A Cauchy Integral Formula for Product Integrals . . . . .	66
2.4 Generalizations . . . . .	77
Notes to Chapter 2 . . . . .	77
<b>Chapter 3 Strong Product Integration . . . . .</b>	<b>79</b>
0 Introduction . . . . .	79
3.1 Direct Extensions of the Results of Chapter 1 . . . . .	80

Cambridge University Press

978-0-521-30230-2 - Product Integration with Applications to Differential Equations

John D. Dollard and Charles N. Friedman

Table of Contents

[More information](#)

viii

Contents

3.2	Generalization . . . . .	83
3.3	The Space $L_s^1(a, b; \mathcal{B}(X))$ . . . . .	86
3.4	Solution of Integral Equations . . . . .	94
3.5	Product Integration of Functions in $L_s^1(a, b)$ . . . . .	98
3.6	Product Integrals Involving Unbounded Operators . . . . .	108
	Notes to Chapter 3 . . . . .	119
<b>Chapter 4</b>	<b>Applications . . . . .</b>	<b>121</b>
4.1	Asymptotics for the Schrödinger Equation . . . . .	121
4.2	Weyl's Limit-Circle Classification . . . . .	132
4.3	The Lie Product Formula . . . . .	135
4.4	The Hille-Yosida Theorem . . . . .	138
4.5	An Example Involving Unbounded Operators with Variable Domain . . . . .	144
	Notes to Chapter 4 . . . . .	148
<b>Chapter 5</b>	<b>Product Integration of Measures . . . . .</b>	<b>151</b>
5.1	Introduction . . . . .	151
5.2	The Product Integral . . . . .	154
5.3	Integral and Differential Equations . . . . .	159
5.4	Further Properties of Product Integrals . . . . .	171
5.5	Improper Product Integration . . . . .	175
5.6	The Schrödinger Equation . . . . .	178
5.7	The Equation $y'' + p(dx)y' + q(dx)y = 0$ . . . . .	182
	Notes to Chapter 5 . . . . .	184
<b>Chapter 6</b>	<b>Complements; Other Work and Further Results on Product Integration . . . . .</b>	<b>187</b>
<b>Appendix I</b>	<b>Matrices . . . . .</b>	<b>193</b>
A.I.1	Elementary Definitions . . . . .	193
A.I.2	Calculus of $C_{n \times n}$ -Valued Functions . . . . .	195
A.I.3	The Canonical Form of a Matrix . . . . .	196
A.I.4	The Spectrum of a Matrix . . . . .	198
A.I.5	Some Additional Results . . . . .	201
<b>References</b>	. . . . .	<b>205</b>
<b>Notes to the References</b>	. . . . .	<b>205</b>

Cambridge University Press

978-0-521-30230-2 - Product Integration with Applications to Differential Equations

John D. Dollard and Charles N. Friedman

Table of Contents

[More information](#)

Contents

ix

**Appendix II by P. R. MASANI:****The Place of Multiplicative Integration in Modern****Analysis** . . . . . 215

A.II.1 Introduction . . . . . 215

A.II.2 Fluid Flows in Smooth Manifolds . . . . . 216

A.II.3 Abstract Formulation of the Theory . . . . . 224

A.II.4 The Evolution Equation in a Pseudolinear Algebra . 225

A.II.5 Linearization . . . . . 226

A.II.6 Discrete-State Markovian Processes with Continuous  
Time Domain . . . . . 228

A.II.7 The Monodromy and Cousin Problems . . . . . 232

A.II.8 The Matricial Hardy and Nevanlinna Classes . . . 237

A.II.9 Holonomy . . . . . 240

A.II.10 Perturbation and Partial Integration . . . . . 245

A.II.11 Concluding Remarks . . . . . 247

References . . . . . 247

Index . . . . . 249