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SINGULAR-PERTURBATION THEORY

AN INTRODUCTION WITH APPLICATIONS

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To

CLOYD VIRGIL SMITH

(1909–1982)

and

THELMA VANANTWERP SMITH

(1910–1983)

IN GRATITUDE

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Preface

A singular-perturbation problem is a problem that depends on a parameter (or parameters) in such a way that solutions behave nonuniformly as the parameter tends toward some limiting value of interest. The nature of the nonuniformity can vary from problem to problem. In practice, one seeks a uniformly valid, easily interpretable approximation to the nonuniformly behaving solution.

Singular-perturbation theory has assumed vast proportions, and this book is not a comprehensive survey of it. I present an introduction to singular-perturbation theory, mostly for ordinary differential equations, with the selection of material conditioned strongly by my own interests. Within the limitations of a single volume it has seemed necessary to omit some topics altogether and to include only brief coverage of some others. For example, there is little coverage of problems for partial differential equations, and none for the singularly perturbed eigenvalue problem or for the abstract Cauchy problem in a Banach space. However, the book does contain points of contact with these and other omitted topics, along with references to the literature, so that an instructor can easily include such omitted topics in a course based on this book. Numerous applications are included from a variety of areas of science and engineering.

This book has evolved from a course of lectures I have given regularly since 1974 at the University of California at San Diego, the Technical University of Munich, and the Mathematics Research Center at the University of Wisconsin at Madison. Versions of the lectures have been given on different occasions to a variety of audiences, including audiences of professional mathematicians and scientists, audiences of graduate students of mathematics and of the pure and applied sciences and engineering, and audiences of advanced undergraduate students.

The book is organized into three main parts, covering various typical classes of singularly perturbed problems for differential equations. Part I deals with oscillation problems of a type arising in many areas, including classical and relativistic mechanics, mathematical astronomy, electric-circuit theory, and biology. Solutions of such problems are quasi-periodic (if damping is not involved) and exhibit certain nonuniformities, for small values of a parameter, as an independent variable ranges over a lengthy interval. Approximate solutions that are valid on such a lengthy interval are constructed here by the method of averaging and by the method of multiple scales.

Part II deals with initial-value problems of nonoscillatory type whose solutions exhibit rapid variation in a thin initial layer. Some typical problems of this type are electric-circuit problems with large resistance and/or small inductance, mechanical problems with small masses and/or large damping, the propagation of radiation through a highly absorbing medium, and chemical and biochemical processes involving simultaneous multiple reactions with widely different reaction rates. Approximate solutions are constructed for such problems here primarily with the O'Malley/Hoppensteadt multivariable method, although a brief discussion of the method of matched asymptotic expansions is included. Part II also includes brief coverage of conditionally stable problems, singular singularly perturbed problems, and numerical methods of singular-perturbation type.

Part III deals with singularly perturbed boundary-value problems of several types, including problems arising in chemical-reactor theory, fluid dynamics, elasticity theory, optimal-control theory, and the physical theory of semiconductors and transistors, as well as nonlinear problems with multiple stable states. Solutions of the boundary-layer type are considered primarily, although solutions of the interior-layer type are also discussed briefly. Approximate solutions are again obtained with a multivariable method. Numerical solutions of such problems are discussed only briefly.

The general approach followed here for problems of every type is to construct a proposed approximate solution and then linearize the original problem about that intended approximate solution. The resulting linearization, which is itself generally a nonlinear problem, is then studied in order to obtain an existence and local-uniqueness result for the original problem, along with useful error estimates for the difference between the resulting exact solution and the given approximate solution. There are several approaches available for study of the linearization, including, but not limited to, the fixed-point method and the method of differential inequalities. Which approach one employs is often a matter of personal taste. I emphasize primarily the Banach/Picard fixed-point method, although I briefly discuss various other approaches as well. Here, as elsewhere, the book allows for flexibility, so that an instructor can emphasize a different approach (or approaches) as preferred.

There is enough material in the book for a one-year course, although I have generally given shorter courses of a semester or even a quarter. The book allows an instructor considerable flexibility in choosing topics to be included or excluded, so that the book can be used for many different courses. I have often omitted portions of the book altogether and skimmed over other portions lightly in different courses, depending on the interests of the audience and the length of the course. Naturally, a one-quarter course would require the instructor to exercise the greatest selectivity.

I hope the book will be useful to students of science and engineering at the graduate and advanced undergraduate levels. The interest inherent in the many applications of the subject makes the book useful not only as an introduction to singular-perturbation theory for differential equations but also as a vehicle by which the student can obtain a broader appreciation of certain important and useful results and procedures from classical and modern analysis.

Del Mar, California Donald R. Smith

Acknowledgments

This book is my presentation of the work of many people, and it is a pleasure for me to acknowledge their work here. Many authors were instrumental in the early development of the study of differential equations and in the use of differential equations to construct mathematical models of natural phenomena, including Isaac Newton (1642–1727), Gottfried Leibniz (1646–1716), Jakob Bernoulli (1654–1705), Johann Bernoulli (1667–1748), Leonhard Euler (1707–83), Joseph-Louis Lagrange (1736–1813), Augustin Cauchy (1789–1857), and Henri Poincaré (1854–1912). Perturbation theory for differential equations is a vast and growing subject.

I have had to be very selective in choosing topics to include in this single volume. The list of references at the end of the book includes some 300 listings, only a fraction of the literature on the subject. Throughout the book I have acknowledged the authors of the work discussed here, as far as I know of them. It is likely that I have, out of ignorance, slighted some authors who should have been mentioned or mentioned more prominently. I ask their forgiveness.

My general dependence on other authors, even to the limited extent to which I am aware of it, is too vast for complete citation here. Of the many recent authors responsible for the work discussed in this book, I mention here only Robert E. O'Malley, Jr. His importance to this book is reflected in the fact that some 10 percent of the references bear his name. In particular, O'Malley is responsible for much of the work discussed in Part II and Part III.

I wish to thank R. James Milgram of the Department of Mathematics at Stanford University for creating and making available to me TECPRINT, a technical word processor that contributed greatly in the writing of this book. The staff of Cambridge University Press did an outstanding job in seeing the book through press; I wish especially to thank David Tranah and Michael Gnat for their many contributions to this project. The copy editor, Jim Mobley, did an exceptionally fine job, for which I am grateful.

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D. R. S.