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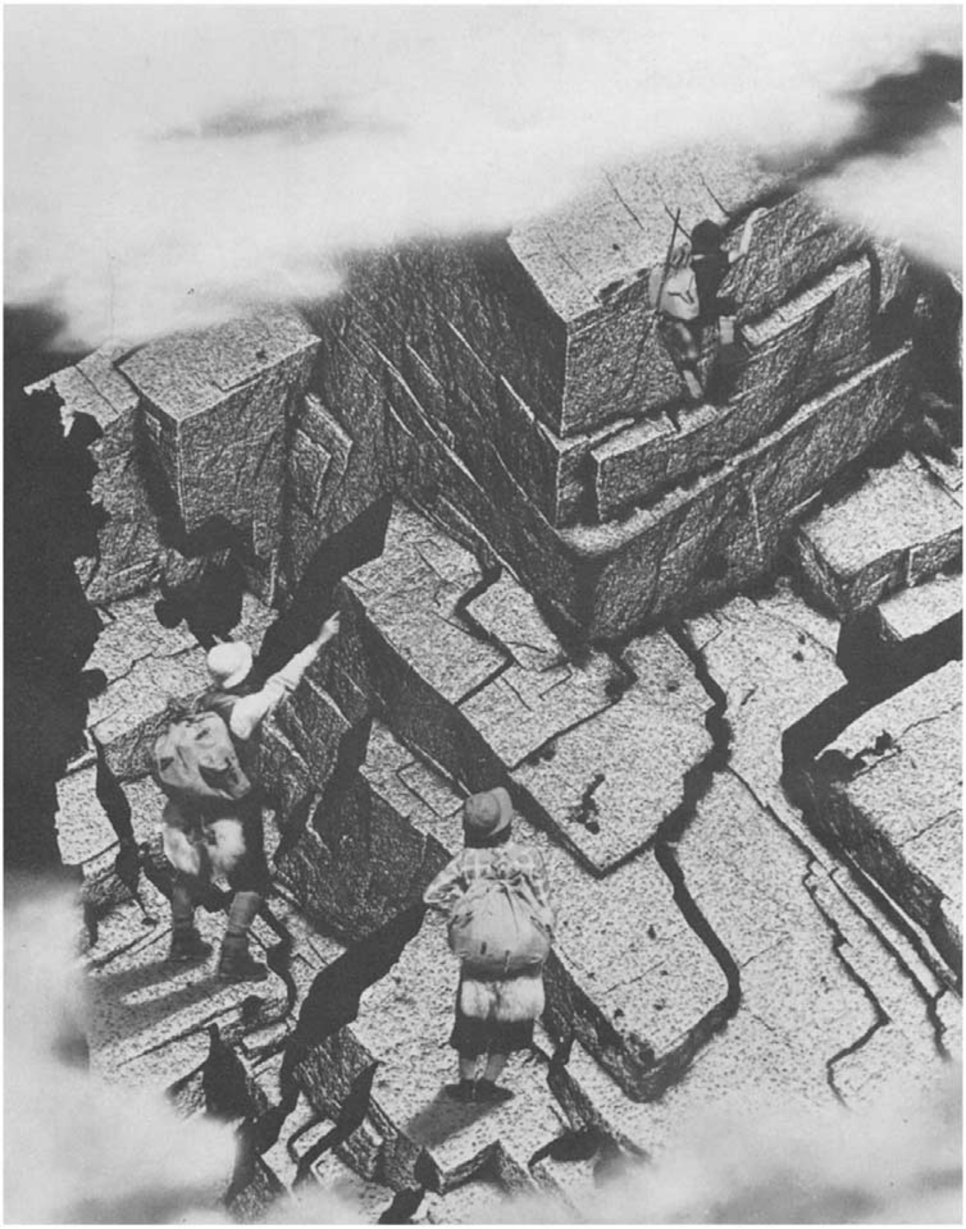

The 'mountain' is the highly magnified surface of a crystal of aluminium; magnification × 35 000. (From *The World through the Electron Microscope*, by courtesy of JEOL Ltd.)

Surfaces

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Second edition

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To the memory of Joe Griffiths (1964–70), who loved to make models of all kinds, but especially paper Origami ones.

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Contents

Preface to the first edition

Over several years, I have tried to teach topics from the geometry of surfaces to various audiences of adults. These varied in technical skill, from Extra-Mural students with plenty of interest but only old-fashioned arithmetic, to Honours mathematics students with A-level skills and attitudes. Almost all students, however, seemed to find difficulty in thinking about spatial objects, and there was little that they could read, to help them; for the conventional developments of the theory are usually found in advanced texts, and often not in English.

It is also nowadays quite possible to pass public examinations in mathematics while avoiding any 3-dimensional problems and the consequent need for sketching. We then have the cycle: children uneducated in 3-dimensional thinking, become teachers without skill in 3-dimensional thinking, who then leave their pupils uneducated in such thinking, who then become teachers . . .

This book is an attempt to break that cycle, being designed for students who might later teach children. It has two aspects, one mathematical, and one educational. Mathematically, it expounds the topology of compact surfaces as far as the Classification Theorem, and treats also the Morse Theory of such surfaces. In fact the whole treatment is a 'handle–body' approach. However, a conventional mathematical treatment would begin 'logically' with topological spaces, continuity and homeomorphisms, before settling down to combinatorial ideas. Any student who could not jump the initial conceptual thresholds would then be cut off from the rich intuitive material that underlies the later work. And it is the rich intuitive material that a future teacher will need, for introducing children to 3-dimensional thinking, so that he can stress the *meaning* rather than the syntax. (The importance of meaning, as against syntax, is emphasized by René Thom in his lecture to the 1972 International Congress on Mathematical Education: see the Proceedings (*Developments in Mathematical Education*, edited by A. G. Howson, Cambridge University Press, 1973), especially p. 206.)

Consequently, this book is written within the discipline of Mathematics Education, and along the lines suggested in my Invited Address to the International Congress of Mathematicians, Nice, 1970. Certain 'Pedagogical Axioms' are assumed at the outset (and given here the more pleasant names of 'Agreements') and a theory is developed from them. The development uses a language which departs from conventional English as little as possible, but which is isomorphic to a 'pukka' mathematical language.

Indeed, in the course of the development, we *introduce* the notions of definition, theorem and proof, and especially the method of mathematical induction. All these ideas are basically new to the typical student who has covered a traditional A-level syllabus in mathematics, even though he may have heard the words. It is unfortunate that many mathematics lecturers assume that because the student has heard the words, they are a part of his being; and that therefore he will immediately comprehend a lecture course of the austere 'Definition, Theorem, Proof' kind without ever knowing why mathematicians invented such a style. Such professional mathematicians will perhaps appreciate the point of view of this book (even if they do not sympathize with it), if I say that it is written as a corrective to the young man who, having been asked to provide a course on surfaces, lectured on the combinatorial theory of n-manifolds, and in the last lecture said, 'Now put $n = 2$, and we get, trivially . . .'. (The adolescent word 'trivially' is not used in our treatment.) The point is that the young man was interested only in getting the mathematics straight in his own head, not with communicating with an audience and taking into account its mathematical skills and understanding. It is when the mode of communication relates the mathematics to such constraints that we meet the discipline of *mathematics education*. Of course, had the young man's constraints included one that said all his audience were future Ph.D. candidates in mathematics, his own curriculum might have been suitable.

For any reader who may have passed the conceptual thresholds mentioned above, I have included Appendix A, containing an outline of a full mathematical treatment, in which (of course) the Pedagogical Axioms are modelled and proved within conventional topology.

In fact, our treatment is related to Topology, as Synthetic is related to Analytic Geometry; the undefined terms of Euclidean Geometry correspond to real-life points, lines etc., but the development of Euclidean Geometry can be modelled isomorphically within Analytic (Coordinate) Geometry. One does not need to understand the construction of the real number system from the integers in order to grasp the theorem of the Nine-Point Circle, for example, or Pascal's Theorem in Projective Geometry. In the same way, the 'panels' and 'paper surfaces' of our treatment have real-life counterparts, and their mathematics can be discussed in a language familiar to beginners. The material of Appendix A might well be discussed in a course for Final Honours undergraduates; but the bulk of the text is designed for less advanced students. A second Appendix consists of 'Teaching Notes', in which reasons are given for adopting certain strategies (marked with a symbol Ⓣ in the text) – why, for example, we might avoid saying that something is an equivalence relation (because it might involve a student in a large effort of understanding, without a compensatory gain in his geometrical insight). Naturally, if a student already understands the notion of equivalence relation, this understanding can be reinforced when he* meets another example of it, especially if *he* points it out rather than being told.

My hope is that some teachers (actual or potential) will use some of the material of the book for teaching students of various ages. Their professional judgement will tell them which topic is suitable for which student, and they

* Throughout, 'he' means 'he or she' when we refer either to teacher or to pupil.

will pick the conversational style that is best suited to communicate with him. For example, now that in Britain abolition of the 'eleven-plus' examination allows imaginative mathematics to be regarded as the norm in our primary schools, young children might model specific surfaces and work out their Euler numbers. In secondary schools, children can work at that part of the material requiring algebra of the form '$\chi = n$' rather than '$\chi = -21$'. It seems to be necessary to encourage habits of modelling and drawing fairly early, because undergraduates seem to think it undignified to begin practising at their late age. I hope, too, that these studies can be associated with geometrical drawing, and painting and sculpture; and especially that by visits to art galleries, objects may be touched and sensed in the round, rather than being diagrammed or photographed. In this way, pupils may be led to speak their own thoughts on geometry, and not merely be told the thoughts of others. By rational discussion they may be led to appreciate both the civilizing notion of proof and the practical benefits that earlier geometry has brought us in engineering, architecture and science.

The only technical prerequisites needed of the reader are that he can follow a diagram, and that he is not afraid of simple algebraic expressions. In this connection, he should understand that if we have a set containing n objects, then we can refer to them as $P_1, P_2, \ldots, P_n$ (short for the object labelled '1', object labelled '2', and so on up to the object labelled 'n'), and read 'P sub 1, P sub 2, etc., up to P sub n'. A few references are made in the text to other books that may be consulted; for example Hilbert and Cohn-Vossen [9] refers to the book listed as number 9 in the references at the end.

Of course, the reader needs something else, which he as a teacher may possess, but which his intended students may not. I refer to curiosity and the wish to know, as well as the patience and stamina necessary to find answers in a systematic way. It should not be assumed (although it often is, in institutions of 'higher' education) that pupils possess such qualities: certainly young children have curiosity, but pressures of their environment may well discourage patience, and emphasize quick returns. Consequently it is part of a teacher's job to encourage these qualities in pupils, and I hope that I have at least provided some interesting material to help in this encouragement.

Many exercises are included, in the hope that the reader will try them, to increase his insight. The more technical ones are indicated by a star, and are not essential for an understanding of the text; but the reader should not be scared by their symbolism, and he should come back to them when his desire to know inspires him to penetrate the algebra. Some hints and solutions are to be found in Appendix C, but some exercises are designed as 'investigations' and it would spoil them if one commented too much; this is why not all exercises are supplied with hints *or* solutions.

I wish here to thank several friends who made helpful comments on a preliminary draft of the book, and gave warm encouragement. Especially, too, I wish to thank the members of the Cambridge University Press for their advice, and their artist who drew the diagrams.

H.B.G.

Southampton, May, 1974

Preface to the second edition

Several reviewers and correspondents have shown a warm interest in the book and made suggestions for improving it; where possible I have altered the original text accordingly. The principal technical problem has been to give more explanation for identifying a twisted bridge, enabling certain proofs to be clarified or corrected. Sometimes I have been asked for fuller commentary, or for greater precision, especially when first introducing an idea, but here I have not always acceded to the request: my aim has been to allow beginners to get to the heart of the matter without being obstructed by too many written words or minor exceptions. (In a classroom, explanations and exceptions can be given conversationally, but they might well freeze if set down on a page.) Thus, some critics have not fully appreciated the philosophy of my earlier Preface and the Teaching Notes, and their criticism has been to demand that the exposition should have the finished look of conventional contemporary mathematics with every objection prepared for in advance. My point here has been to *generate* objections, in order to create a wish in students for greater precision in language and rigour *once they have some interesting mathematics to care about*. This repeats a historical process so beautifully captured by Lakatos, in his book *Proofs and Refutations* (Cambridge University Press, 1976), where his principal example is the development of Euler's Theorem ($\chi(\text{sphere}) = 2$) – although his thesis is exemplified in Analysis just as well. The reader interested in a fuller discussion of the philosophy behind the present exposition may care to read the interchange between R. Schwarzenberger and myself in the *Mathematical Gazette* (**60**, 1976).

Particular thanks are due to Benno Artmann and Gwylym Edmunds, for their comments after using the text with students; Gwylym kindly allowed me to use his tutorial sheets and examination papers on which to base the extended set of Exercises forming the newly-added Chapter 9. I am also grateful to Tony Gardiner and John Rigby for sending me detailed letters of criticism, and to the translators of the French and German editions for raising fruitful expository points and finding errors. Once again, members of Cambridge University Press gave me advice and help with their usual high standard of expertise and kindness.

H.B.G.

 Southampton, 1980