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978-0-521-29930-5 - The Foundations of Analysis: A Straightforward Introduction

K. G. Binmore

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†This material is more advanced than the remaining material and can be omitted at a first reading.

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