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978-0-521-29930-5 - The Foundations of Analysis: A Straightforward Introduction

K. G. Binmore

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**THE FOUNDATIONS OF ANALYSIS:
A STRAIGHTFORWARD INTRODUCTION**

**BOOK 2
TOPOLOGICAL IDEAS**

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THE FOUNDATIONS OF
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INTRODUCTION

BOOK 2
TOPOLOGICAL IDEAS

K. G. BINMORE

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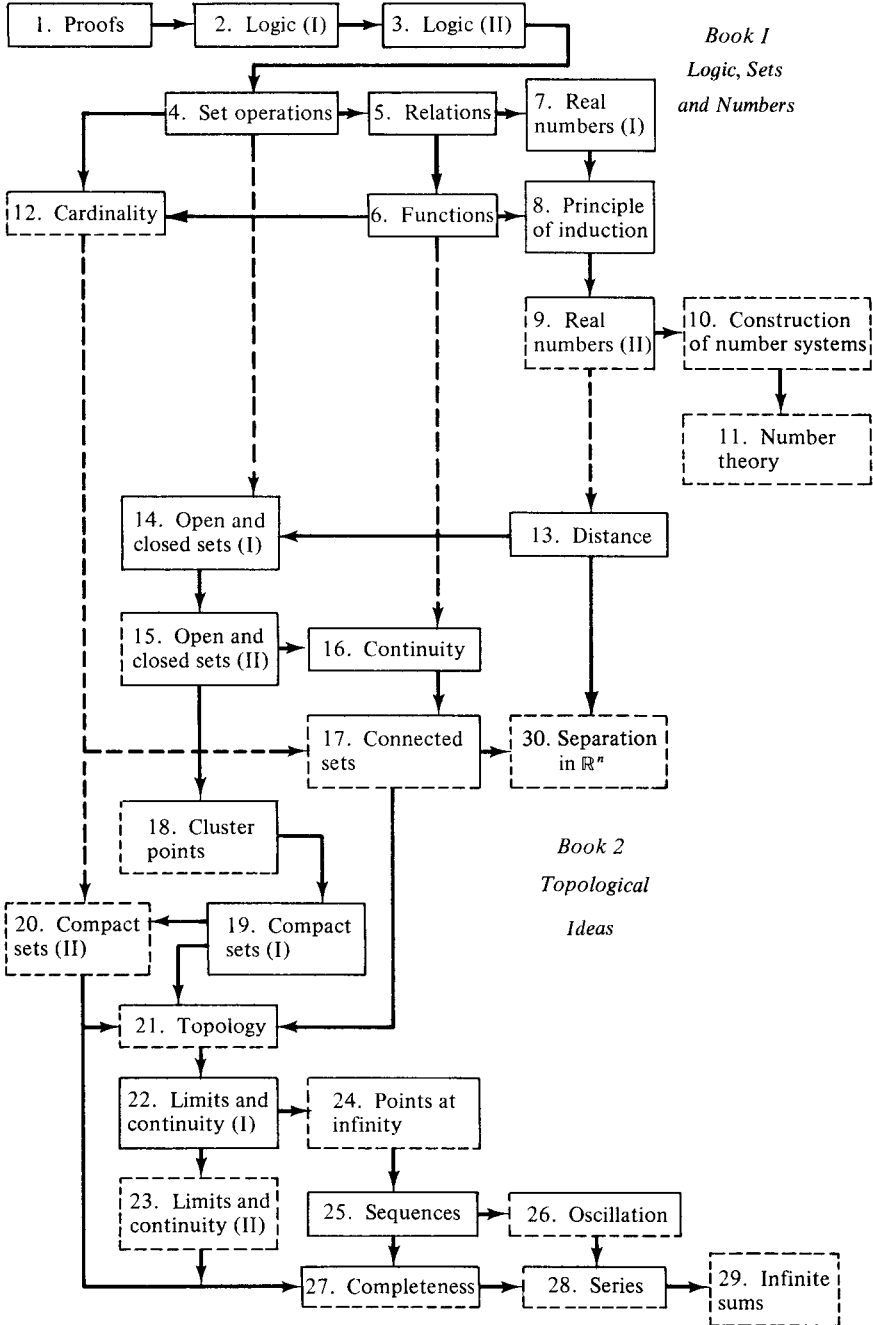
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The diagram on p. x illustrates the logical structure of the books. Broken lines enclosing a chapter heading indicate more advanced material which can be omitted at a first reading. The second book depends only to a limited extent on the first. The broken arrows indicate the extent of this dependence. It will be apparent that those with some previous knowledge of elementary abstract algebra will be in a position to tackle the second book without necessarily having read the first.



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INTRODUCTION

This book is intended to bridge the gap between introductory texts in mathematical analysis and more advanced texts dealing with real and complex analysis, functional analysis and general topology. The discontinuity in the level of sophistication adopted in the introductory books as compared with the more advanced works can often represent a serious handicap to students of the subject especially if their grasp of the elementary material is not as firm as perhaps it might be. In this volume, considerable pains have been taken to introduce new ideas slowly and systematically and to relate these ideas carefully to earlier work in the knowledge that this earlier work will not always have been fully assimilated. The object is therefore not only to cover new ground in readiness for more advanced work but also to illuminate and to unify the work which will have been covered already.

Topological ideas readily admit a succinct and elegant abstract exposition. But I have found it wiser to adopt a more prosaic and leisurely approach firmly wedded to applications in the space \mathbb{R}^n . The idea of a relative topology, for example, is one which always seems to cause distress if introduced prematurely.

The first nine chapters of this book are concerned with open and closed sets, continuity, compactness and connectedness in metric spaces (with some fleeting references to topological spaces) but virtually all examples are drawn from \mathbb{R}^n . These ideas are developed independently of the notion of a limit so that this can then be subsequently introduced at a fairly high level of generality. My experience is that all students appreciate the rest from 'epsilonese' made possible by this arrangement and that many students who do not fully understand the significance of a limiting process as first explained find the presentation of the same concept in a fairly abstract setting very illuminating provided that some effort is taken to relate the abstract definition to the more concrete examples they have met before. The notion of a limit is, of course, the single most important concept in mathematical analysis. The remainder of the volume is therefore largely devoted to the application of this idea in various important special cases.

Much of the content of this book will be accessible to undergraduate students during the second half of their first year of study. This material has

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been indicated by the use of a larger typeface than that used for the more advanced material (which has been further distinguished by the use of the symbol †). There can be few institutions, however, with sufficient teaching time available to allow all the material theoretically accessible to first year students actually to be taught in their first year. Most students will therefore encounter the bulk of the work presented in this volume in their second or later years of study.

Those reading the book independently of a taught course would be wise to leave the more advanced sections (smaller typeface and marked with a †) for a second reading. This applies also to those who read the book during the long vacation separating their first and second years at an institute of higher education. Note, incidentally, that the exercises are intended as an integral part of the text. In general there is little point in seeking to read a mathematics book unless one simultaneously attempts a substantial number of the exercises given.

This is the second of two books with the common umbrella title *Foundations of Analysis: A Straightforward Introduction*. The first of these two books, subtitled *Logic, Sets and Numbers* covers the set theoretic and algebraic foundations of the subject. But those with some knowledge of elementary abstract algebra will find that *Topological Ideas* can be read without the need for a preliminary reading of *Logic, Sets and Numbers* (although I hope that most readers will think it worthwhile to acquire both).

A suitable preparation for both books is the author's introductory text, *Mathematical Analysis: A Straightforward Approach*. There is a small overlap in content between this introductory book and *Topological Ideas* in order that the latter work may be read without reference to the former.

Finally, I would like to express my gratitude to Mimi Bell for typing the manuscript with such indefatigable patience. My thanks also go to the students of L.S.E. on whom I have experimented with various types of exposition over the years. I have always found them to be a lively and appreciative audience and this book owes a good deal to their contributions.

June 1980

K. G. BINMORE