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† This material is more advanced than the main body of the text and is perhaps best omitted at a first reading.

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K. G. Binmore

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