THE FOUNDATIONS OF ANALYSIS: A STRAIGHTFORWARD INTRODUCTION

Book 1
Logic, Sets and Numbers

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CONTENTS

†This material is more advanced than the main body of the text and is perhaps best omitted at a first reading.

INTRODUCTION

This book contains an informal but systematic account of the logical and algebraic foundations of mathematical analysis written at a fairly elementary level. The book is entirely self-contained but will be most useful to students who have already taken, or are in the process of taking, an introductory course in basic mathematical analysis. Such a course necessarily concentrates on the notion of convergence and the rudiments of the differential and integral calculus. Little time is therefore left for consideration of the foundations of the subject. But the foundational issues are too important to be neglected or to be left entirely in the hands of the algebraists (whose views on what is important do not always coincide with those of an analyst). In particular, a good grasp of the material treated in this book is essential as a basis for more advanced work in analysis. The fact remains, however, that a quart will not fit into a pint bottle and only so many topics can be covered in a given number of lectures. In my own lecture course I deal with this problem to some extent by encouraging students to read the more elementary material covered in this book for themselves, monitoring their progress through problem classes. This seems to work quite well and it is for this reason that substantial sections of the text have been written with a view to facilitating 'self-study', even though this leads to a certain amount of repetition and of discussion of topics which some readers will find very elementary. Readers are invited to skip rather briskly through these sections if at all possible.

This is the first of two books with the common title

Foundations of Analysis: A Straightforward Introduction.

The current book, subtitled

Logic, Sets and Numbers,

was conceived as an introduction to the second book, subtitled

Topological Ideas

and as a companion to the author's previous book

Mathematical Analysis: A Straightforward Approach.

Although *Logic, Sets and Numbers* may profitably be read independently of these other books, I hope that some teachers will wish to use the three books together as a basis for a sequence of lectures to be given in the first two years of a mathematics degree.

Certain sections of the book have been marked with a † and printed in smaller type. This indicates material which, although relevant and interesting, I regard as unsuitable for inclusion in a first year analysis course, usually because it is too advanced, or else because it is better taught as part of an algebra course. This fact is reflected in the style of exposition adopted in these sections, much more being left to the reader than in the body of the text. Occasionally, on such topics as Zermelo–Fraenkel set theory or transfinite arithmetic, only a brief indication of the general ideas is attempted. Those reading the book independently of a taught course would be wise to leave those sections marked with a † for a second reading.

A substantial number of exercises have been provided and these should be regarded as an integral part of the text. The exercises are not intended as intelligence tests. By and large they require little in the way of ingenuity. and, in any case, a large number of hints are given. The purpose of the exercises is to give the reader an opportunity to test his or her understanding of the text. Mathematical concepts are sometimes considerably more subtle than they seem at first sight and it is often not until one has failed to solve some straightforward exercises based on a particular concept that one begins to realise that this is the case.

Finally, I would like to thank Mimi Bell for typing the manuscript for me so carefully and patiently.

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