AN INTRODUCTION TO ABSTRACT ALGEBRA

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VOLUME 2

BY
F. M. HALL
Head of the Mathematics Faculty
Shrewsbury School

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PREFACE

This book is the second of a two volume work which attempts to give a broad introduction to the subject of abstract algebra. It assumes no previous knowledge of this work but does expect a fair amount of mathematical sophistication. Thus it is not intended to be used as a textbook at an elementary level in schools, as part of one of the 'modern mathematics' courses, but is aimed rather at a fairly intelligent sixth-former, who has been brought up on either traditional or modern syllabuses and who wishes to know something more about modern algebra, for example as a preparation for university work. This second volume in particular should be found useful by undergraduates, as giving a broad background before a study in more depth is undertaken, and it does in fact cover a great deal of work usually included in university first degree courses, though not all such work. Teachers in schools and students in teacher training colleges who wish to learn some abstract algebra as an aid and background to their teaching should also find parts of both volumes interesting and useful. Numerous books on the subject have been and are being written, but these are usually either too advanced, with a lack of motivation, for the beginner, or are written at a far less sophisticated level, for younger pupils. It is still not easy to find many which start from the beginning and lead up to substantial and important ideas gradually and in a fairly elementary manner.

Volume 1 dealt with various special sets, such as the integers, residue classes and polynomials, emphasising their structure before discussing the fundamental laws of algebra and finishing with three chapters on groups, carrying the theory as far as Lagrange's theorem.

The second volume is arranged so that it may be read independently of the first, and the first chapter gives a shortened account of the group theory which was included in volume 1. Thus it is not necessary for a student to have read volume 1 previously, and many university undergraduates may well use the present book by itself. A reader with a scant knowledge of structure would be advised however to look through at least part (if not all) of the first volume, and such a reader may omit chapter 1.

The first chapter is followed by essential work on group-homomorphisms, and we then consider rings, fields and integral domains. The work towards the end of chapter 4, and some in chapter 5, is quite advanced though explained as simply as possible. Chapter 6 continues the study of group theory by considering the vital concept of an invariant subgroup, and the next chapter introduces the related idea of an ideal in a ring. This chapter, and the following one, deals with quite deep properties, and the reader may well find them heavy going.

With chapter 9 we are on much simpler ground, and give a short account of the theory of vector spaces, from the point of view of abstract structure rather than as an adjunct to matrix theory, although we briefly introduce the idea of matrix as a representation of a linear transformation. Chapter 10 applies the vector space idea to the axiomatic foundations of geometry, and we conclude the formal work of the book by a fairly abstract account of Boolean algebra. The final chapter is informal, and indicates how some of the work may be further developed.

As in the first volume, I have been careful to give as much explanation as possible about the reasons for doing the various topics, and have explained the methods carefully. The difficulty which many students experience arises usually from their not understanding the motivation of the work as a whole: they do not see where it is leading. I have tried to avoid this sense of frustration by introducing new ideas carefully and sometimes, as in the case of invariant subgroups, at some length. The proofs are nearly always given in full detail and are selected for ease of understanding rather than conciseness. Concrete examples of new structures are given as much as possible, though inevitably they are usually drawn from mathematics itself, and are often necessarily rather more abstract than was the case in the first volume. But of course we meet again many of the particular sets that were studied in that volume, and they are now revealed in their true structural form (e.g. polynomials occur in many places in this volume).

The book may be read with little or no aid from a teacher, and each chapter except the last ends with a few worked exercises. The exercises themselves are, as in volume 1, divided into A and B: the first are quite straightforward and should be worked completely. The B exercises are very variable; some are fairly straightforward, others quite difficult and a few give extensions of the bookwork. The reader is not expected to be able to do all these exercises, at least not at a first reading.

I would like to thank a former pupil at Dulwich College, Mr J. R. Pratt, who read the manuscript and made many valuable suggestions. I am indebted to my colleague, Mr S. D. Baxter, who read the proofs; and am grateful to the Cambridge University Press for their help throughout all stages of the preparation of the book.

F.M.H.

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